Legalization of Prostitution and its Impact on the Market for Human Trafficking

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LEGALIZATION OF PROSTITUTION
AND ITS IMPACT ON THE MARKET
FOR HUMAN TRAFFICKING

INDEPENDENT STUDY THESIS

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by

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Abstract

Human trafficking is a form of modern slavery which involves the illegal trade of humans for the purpose of forced labour and/or sexual exploitation. Most victims of human trafficking are women and children trafficked for the purpose of sexual exploitation. Thus, the legal framework regarding prostitution may heavily influence the profitability of trafficking and the inflow of trafficked victims to a given country. This paper uses a compartmental epidemiological model to create a theoretical framework around the market for human trafficking and shows that the legalization of prostitution will lead to an increase in the volume of trafficked victims.
This work is dedicated to the future generations of Wooster students.
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CHAPTER 1

INTRODUCTION

Human trafficking is a form of modern slavery in which humans are forcefully traded. [28] According to the International Labour Organization, at any given point in time, 3 in every 1000 people are being trafficked in the world [2]. The Global Report on Trafficking in Persons (2014) published by the United Nations Office on Drugs and Crime shows a sharp increase in the reported cases of trafficking over the last few decades [39] thus, making this a matter of global importance. Trafficking in human beings is considered to be a serious violation of human rights and its reduction is highly prioritized in many countries [28]. Trafficking is mostly done for the purpose of forced labour or commercial sexual exploitation. In this study, we focus our attention on sex trafficking, i.e., trafficking done for the purpose of prostitution.

Prostitution has now become a multi-billion dollar, growing industry [44]. Most commercial sex workers are estimated to be women and girls, with the majority of demand arising from male customers [21]. Although the debate surrounding the legalization of prostitution has been around for decades, more and more countries in the world are now seen to be legalizing the occupation. The proponents of legalization argue that the legalization of prostitution would allow for the protection of sex workers, reduce violence and improve working conditions by allowing the industry to be regulated by the government [42]. For example, the
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Netherlands lifted the ban on brothels in 2000 to ‘sever the links between prostitution and crime, protect minors against sexual abuse and protect the position of sex workers’ among other objectives [6]. The opponents of prostitution however, argue that legalization has worsened the situation for workers, as it has simply expanded the sex industry, instead of protecting it. This has also lead to a major demotion of women’s health [23].

The debate surrounding the legal status of prostitution is an on-going discussion, but a few key consequences of legalization particularly seem to attract more attention. First, the legalization of prostitution significantly decreases a brothel owner’s risk of employing trafficked victims, as they can now hide their activities under the pretense of the legal employment of prostitutes. This increases a brothel owner’s incentive to employ trafficked prostitutes. Second, the legalization of prostitution causes a general increase in the demand for prostitution services, as more customers look to enter the industry due to removal of the risk of criminal prosecution. This provides an incentive for more suppliers or brothel owners to enter the industry, thereby causing a general increase in the supply of prostitution as well.

It is also known fact that brothel owners around the world pay only a small sum of money or usually no money, to trafficked prostitutes [29]. This makes trafficked prostitutes a cheap labour option in comparison to voluntary prostitutes. In addition, it is often found that customers prefer services provided by a variety of women from different nationalities [29] [23]. This motivates brothel owners to use trafficked victims to satisfy consumer demands, as it usually implies a higher price paid by customers [9]. All of these reasons lead us to theorize that the demand for trafficked victims outweighs the demand for voluntary prostitutes. Thus, we hypothesize that the legalization of prostitution will lead to an increase in the number of individuals trafficked.
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In order to show this relationship, we use a compartmental epidemiological system to analyze the dynamics of the sex trafficking industry, as a model of demand and supply. The structure of the study is as follows: Chapter 2 provides a theoretical framework that defines human trafficking and discusses the relationship between the legal status of prostitution and individuals trafficked for sexual exploitation. This is followed by a review of empirical and theoretical research carried out on this subject in Chapter 3. In Chapter 4, we explore some of the theory used by the studies reviewed in Chapter 3, in order to be able to develop the tools needed to create a working model. In Chapter 5, we numerically simulate and modify a model taken from Davidoff et al (2006) as discussed in Chapter 3. We then create a realistic base model that includes the element of human trafficking, and use it to perform numerical simulations to analyze the effects that the legalization of prostitution has on trafficked individuals. An increase in the level of trafficked victims employed in the prostitution industry is anticipated as a result of legalized prostitution. This is confirmed by the numerical simulations carried out in Chapter 6. Finally, we conclude in Chapter 7 with a discussion of our results and the limitations that need to be addressed in our model for the purpose of future research on the subject.

This study asserts that the demand for prostitution services has a major impact on the supply of prostitutes, although the reverse is not seen to be true. Curbing the demand for prostitution through stricter regulation imposed on customers, goes a long way in reducing the level of prostitutes in the industry, however, it is not enough to significantly decrease the volume of sex trafficking involved. Trafficking is found to reduce only if the demand for commercial sex is completely removed or if barriers to entry remove the incentive for brothel owners to employ trafficked victims, as they now represent a more expensive labour cost. Thus, it is recommended that future research on this topic be focused on the characteristics of the recruitment rate of trafficked individuals.
HUMAN TRAFFICKING: AN INTRODUCTION

In this chapter we introduce the reader to the inner workings of the world of prostitution, exploring the definitions of human trafficking, why it occurs, the trafficking process, and the reason for its continuous occurrence. We also look at the changes in the prostitution industry under a system of legalization. The structure and discussion of this chapter is largely based on Kara (2009) [29].

2.1 PROSTITUTION

Often deemed as one of the world’s oldest occupations, prostitution is a global phenomenon. According to the website, US Legal, prostitution is defined as the “the commission by a person of any natural or unnatural sexual act, deviate sexual intercourse, or sexual contact for monetary consideration or [some] other thing of value” [31]. A prostitute is therefore any person who provides sexual services in exchange for money or some other good. In most countries, the involvement in prostitution is looked upon as a social evil, with prostitutes usually being ostracized from society. Additionally, it is usually the prostitute who is criminalized for providing sexual services instead of the clients who buy such services or brothel owners who employ prostitutes. The method of entry into prostitution is usually categorized into two situations: prostitutes that have voluntarily chosen to be
involved in the industry and prostitutes that have been trafficked into the industry. Voluntary prostitutes are generally seen to enter the industry due to financial pressure, a lack of education and emotional or physical abuse. [43] Trafficked prostitutes on the other hand, are forced into the industry against their own will. Before delving into further detail about prostitution, we will now explore some of the characteristics of the market for human trafficking.

2.2 **What is Human Trafficking?**

Human trafficking is a component of organized crime and a form of modern slavery in which humans are forcefully exploited and traded for various purposes. According to the International Labour Organization, at any given point in time, around 2.5 million people are being trafficked in the world [2]. Empirical evidence shows a sharp increase in the reported cases of trafficking over the last few decades, making this a matter of global importance [39]. Trafficking in human beings is considered to be a serious violation of human rights and its reduction is highly prioritized in many countries. The 2000 United Nations Trafficking Protocol defined trafficking as: “the recruitment, transportation, transfer, harbouring or receipt of persons, by means of threat or use of force or other forms of coercion, of abduction, of fraud, of deception, of the abuse of power or of a position of vulnerability or of the giving or receiving of payments or benefits to achieve the consent of a person having control over another person, for the purpose of exploitation. Exploitation shall include, at a minimum, the exploitation of the prostitution of others or other forms of sexual exploitation, forced labour or services, slavery or practices similar to slavery, servitude or the removal, manipulation or implantation of organs” [4].

Trafficking is done mostly for the purpose of forced labour or commercial sexual exploitation [4]. In this chapter, we focus our attention on the latter. Sex trafficking
involves violence, threats, and debt bondage to compel victims to engage in acts of commercial sex against their will. Sex trafficking victims around the world usually involve women and children. All cases of sex trafficking involve three main steps: recruitment, movement and exploitation [29] [10] [41].

2.2.1 Recruitment of Sex Slaves

Among the various methods of recruitment, we discuss four primary avenues: poverty-induced circumstances, abduction, seduction or false marriage proposals, and recruitment by former slaves.

2.2.1.1 Poverty-Induced Circumstances

Poverty is one of the biggest driving forces for sex trafficking and make women easily vulnerable to deceit and recruitment into the trade. When individuals are under extreme poverty and are desperate due to a lack of economic opportunities, traffickers take advantage of such situations by offering these individuals false jobs and making them fake promises of a better life in a rich nation. For example, Kara (2009) [29] stated that newspapers such as Makler in Moldova are known for posting false jobs offers through which thousands of victims are known to have been trafficked to countries in Asia and Europe. Even though people know that many such vocational offers are false, they still accept them due to their lack of economic opportunity in the hopes that “nothing bad will happen to me”[29]. Nigeria is another giant recruitment center for trafficked victims. One of the poorest countries in the world today, most Nigerian individuals are characterized as being highly susceptible, uneducated, and without any economic opportunity to better their circumstances or provide for their families. This makes Nigeria highly appealing for traffickers looking to take advantage of the dire vulnerability facing the women in the country.
Another effective location to recruit slaves through deceit is in refugee camps, with approximately 33 million people displaced due to civil war and other disasters according to an estimate [29]. In most cases, refugees are not allowed to leave or seek any employment in their host countries. This provides traffickers a perfect opportunity to recruit individuals who are simply looking for a means of survival and a chance of improving their lives.

Another dismal reality of such poverty induced circumstances involves families willingly selling their children in order to make ends meet [41] [10]. In such situations, families are almost forced to make a decision to sell their child for a price as low as twenty to thirty dollars. In addition to the price of the child, slave traders send back monthly wages to the families of these victims in order to entice more families to sell their children. In an interview conducted with a young trafficked victim called Bridgitte, it was clear that most victims of trafficking are forced to remain in sex slavery for many years due to the money sent back to their families. In such situations, these victims have said to feel like “slot machines to our families” [29].

2.2.1.2 ABDUCTION

Contrary to what is portrayed by the media, abduction is one of the less popular methods of recruiting sex slaves. This is due to a variety of reasons involving long commutes to destination countries and trafficked victims being likely to escape at any given opportunity. Nonetheless, there still remain a few cases of abduction that render it viable as a means of recruitment by slave traders.

2.2.1.3 SEDUCTION OR RECRUITMENT THROUGH FALSE MARRIAGE PROPOSALS

In many countries, women are subjected to a historic, deeply rooted oppression, where the only way to escape ostracism, disrespect and inequality and gain basic
rights and social acceptance is through the prospect of getting a husband [29]. In such countries where gender bias renders life extremely difficult for unmarried women, false marriage offers are a particularly lucrative mechanism to recruit women into sexual enslavement. For example, in rural Albania, a former sex slave named Pira narrated that she was sold by her ‘husband’ to the owners of a sex club on the very night of her marriage. In Nepal, it is very common for slave traders to recruit Indian men “to act as prospective grooms, provide testimonials, have phone conversations and pledge a fairy-tale life” to vulnerable women who are subject to this deceit [29]. In such situations, the young wife is immediately sold to brothels and forced to send handwritten letters to their families to provide a false assurance of a happy marriage.

2.2.1.4 Recruitment Through Former Sex Workers

The role of women as a source of trafficking individuals is deemed to have been “significant and increasing” [10]. The International Organization for Migration’s Counter-Trafficking Module database reports that about 42 percent of the 9,646 recruiters were women. This data represented 78 countries from 1999 to 2006. The United Nations Office on Drugs and Crime (UNODC) reported that 60 percent of the traffickers prosecuted in Nigeria are women and 25 percent of the traffickers prosecuted in the Slovak Republic are women [10].

Recruitment of trafficked victims through former victims is one of the many aspects revealing the complex psychology and harsh reality of sexual enslavement. Years of torture and enslavement turn many women into allies of their recruiters or slave owners. In some cases, women are threatened or given financial incentives to carry out recruitment in their home towns. Most situations involve a case of “happy trafficking” as said by the United Nations. Under this scheme, a few select, trusted trafficked victims are freed for a period of time and sent back to their home
countries and/or villages dressed in expensive garments and usually with a large sum of money to recruit new slaves. These women involved in “happy trafficking” are then usually involved in the creation of a “happy illusion” regarding their life working in a job abroad. This method of recruitment is often considered to be the most useful as it involves minimal effort and usually proves to be an easy way to convince new victims to join the “workforce” [10].

2.2.2 Movement of Trafficked Victims

The movement of trafficked victims usually involves an origin, transit, and destination [29] [10]. In the case of transnational trafficking, bribes are usually paid to border officials and false documents used in order to ensure the smooth transition into the destination country. Trafficked victims are usually recruited from a poor, rural region and transported to a richer, more urban region. During the journey, various methods are involved in order to make the victim easy to transport. For example, a former slave named Sushila was said to be given ice-cream laced with opiates during a three-day bus ride from Nepal to Mumbai. The opiates caused her to sleep for the entire bus journey, only to wake up the morning of her sale. In another case, Nigerian women reported to be raped and tortured while they were in transit so that they could be “groomed and prepared” for the life that awaited them in their destination country [29].

2.2.3 Exploitation of Trafficked Victims

Exploitation is seen by traffickers as a mechanism to ‘break’ the slave in order for her to become more submissive at the time of sale. In this regard, slaves are beaten, sexually assaulted, starved and drugged during the transportation process itself. This exploitation continues even after a victim has been sold, to make certain that
the victim remains submissive and does not try to escape the premises. In some extremely disturbing circumstances, the arms of ‘misbehaved’ minors were broken and their throats cut in front of other sex slaves due to their attempt at escape [29]. Thus, it is extremely difficult for a victim to escape trafficking. In addition, trafficked women are often placed into a system of ‘debt-bondage.’ In this system, victims are expected to pay back the costs of their own trafficking and any other survival costs involved. Trafficked victims however, rarely manage to break out of this debt-bondage system as they are usually unable to repay these costs. This exploitation allows brothel owners to make use of the cheap labour provided by victims and maximize the profits earned by each trafficked worker [18].

2.3 Why Does Human Trafficking Continue To Thrive?

There are many reasons as to why human trafficking has continued to thrive despite global efforts. First and foremost, it is extremely difficult for a victim of sex trafficking to successfully escape. Large trafficking networks and control and influence in the neighbouring regions make escape almost impossible. Escape is even more rare in the case of transnational trafficking. These victims usually are not familiar with their surroundings and more often than not do not even understand the language spoken in the country. In other cases, the system of debt-bondage created by the trafficker and the systematic breakdown of victims ensures the submissiveness of the victim. In these cases, traffickers force victims to continue working until they pay off their ‘debt’ or their purchase price. Threats to the family of these victims are usually made in order to ensure obedience [29] [10] [18].

Second, legal authorities have done very little in the recent past to help with the abolishment of trafficking due to poorly enforced laws and underfunding. There are very few resources allocated to combat sex trafficking as compared to
other forms of trafficking such as narcotics trafficking. This makes it extremely
difficult to identify and penetrate the established trafficking networks in order to
help and rescue victims. Additionally, in most countries, police officials are also
customers of commercial sex with corruption and red tape striking some of the
highest corners of the political structure. To ensure the uninterrupted flow of
business, some traffickers and brothel owners come into agreements with the police
which makes the authorities turn a blind eye towards this industry. Thus, all these
factors combined make it extremely difficult to lower the level of trafficking in a
given country [29][10][12].

2.4 **Legalization of Prostitution**

To theorize the effect that the legalization of prostitution has on the market for human
trafficking, we must discuss factors that go into the economics behind both the
trafficking and prostitution industries. One of the most important factors involved
in human trafficking is the level of risk faced by both traffickers as well as brothel
owners. When prostitution is illegal, the risk faced by brothel owners who employ
trafficked victims falls under two distinct categories. First, there is a risk involved
with supplying prostitutes when prostitution is illegal. Second, there is also a
risk involved with being directly associated in the supply of trafficked prostitutes.
When prostitution is legalized, the risk associated with the supply of prostitutes
ceases to exist as prostitution is now considered a legal phenomenon. Thus, the
only risk involved is one that is associated with the supply of trafficked prostitutes.
However, as literature suggests, brothel owners often get away with deceiving legal
authorities into believing that trafficked victims are actually voluntary prostitutes
[23]. In addition, it is often the case that brothel owners have an arrangement with
local legal authorities through the payment of bribes that allow a blind eye to be
turned towards their activities [29]. Thus, this decreased level of risk provides brothel owners with an incentive to employ trafficked workers.

Another reason for brothel owners to employ trafficked victims over voluntary prostitutes is that trafficked prostitutes represent lower labour costs. Research says that brothel owners often pay extremely low to zero wages to trafficked prostitutes [13]. Thus, apart from the initial payment made to traffickers to buy trafficked victims, brothel owners who employ trafficked prostitutes face almost no labour costs. This is also due to the system of debt-bondage established by brothel owners. Thus, brothel owners face lower wage costs when employing trafficked prostitutes than when employing voluntary prostitutes, to whom they would have to pay at least a minimum wage. This in turn provides a greater incentive to brothel owners to employ trafficked victims as it allows for them to enjoy greater profits.

In addition, consumer preferences also play a major role in determining the supply of prostitutes. It is often found that most customers prefer a variety of 'bodies' to choose from as can be seen from the percentage of international sex workers found in each country [23]. For example, Indian men demonstrate a high preference for Nepalese prostitutes rather than Indian prostitutes. Similarly, it is found that the demand for Nigerian prostitutes is extremely high in some European countries, specifically Italy [29]. Thus, brothel owners tend to address these sets of preferences by employing trafficked women as they tend to be easier to recruit than voluntary prostitutes who only enter prostitution due to financial reasons and a lack of other alternative choices [23].

The legalization of prostitution will also lead to an increase in the demand of prostitution services, as customers have an incentive to consume such services due to the disappearance of the risk of arrests. In addition, the social stigma associated with the industry is assumed to have a slight decrease. Therefore to take advantage
of this increase in demand, more suppliers will look to enter the industry, which will further lead to the expansion of the sex trafficking industry.

Thus, in summary, legalization would initially lead to an expansion of the market for sex services. The demand for such services would see a significant increase as will the supply of such services, through both voluntary and trafficked prostitutes. Legalization would attract a certain portion of the population who would join the business of prostitution voluntarily. In addition, legalization would also attract more brothel owners looking to take advantage of the increased demand and to maximize their profits through the employment of of sex slaves. Thus, in theory, the legalization of prostitution will usually lead to an increase in the number of individuals trafficked [36].

In this chapter, we introduced the reader to the market for human trafficking and discussed the different components involved in the functioning of the trafficking market. We also briefly explored the economic effect that the legalization of prostitution would have on human trafficking. We can now begin the development of a model for the trafficking industry. But first, we look at literature involving a compartmental epidemiological model on which our trafficking model will be based.
CHAPTER 3

A REVIEW OF LITERATURE

Chapter 2 comprised of an introduction to prostitution and the world of human trafficking. We delved into the definition of prostitution, the reasons women enter and the industry and the two types of prostitutes found: voluntary prostitutes and trafficked prostitutes. We also introduced the reader to the workings of the human trafficking industry and the impact that the legalization of prostitution may have on it. In this chapter, we review relevant literature, that provides us with a platform to build our model and create a theoretical framework for assessing our hypothesis in Chapter 6.

We discuss two studies that utilize epidemic theory and compartmental epidemiological models to mathematically analyze the dynamics of the prostitution industry. The first study by Davidoff et al (2006) [21] provides a general framework of the forces involved in the market for prostitution and provides policy recommendations in order to curb the number of women in the business. It presents two mathematical models, a female model representing the supply side of prostitution, and a male model representing the demand side of prostitution. It then analyzes both the models and proceeds to explore the coupled system numerically. The analysis concludes with the observation that if law enforcement authorities focused their efforts on making male arrests, the number of women involved in the prostitution business would significantly decrease.
3. A Review of Literature

The second study by Oduwole and Shehu (2013) \([37]\) builds off of the model used by Davidoff et al (2006) \([21]\) and tries to answer a similar question regarding the impact of government intervention on the market for prostitution in Nigeria. However, the study is primarily focused on the impact on prostitution caused by the level of poverty in a country. The results of the study show that poverty and prostitution are directly linked and an increase in government effort to rehabilitate the poor and underprivileged would go a long way in curbing the number of individuals who enter the market for prostitution.

The final study looks at the relationship between legal prostitution and human trafficking by utilizing a multivariate regression model using macroeconomic data. The results of the study validate our hypothesis by indicating that countries with legalized prostitution were more likely to experience larger trafficking inflows than countries where prostitution is illegal.

Before we begin the discussion on the literature, we provide a preview of the workings of general epidemiological models in section 3.1 in order to familiarize the readers with the terminology of epidemic theory used in the later sections of this chapter.

3.1 Preview of an Epidemiological Model

Compartmental epidemiological models are models that divide populations into different compartments depending on certain assumptions and characteristics. It then models the flow of individuals from ‘infected’ compartments to ‘non-infected’ compartments to analyze the spread of a disease. We will explain this concept using a basic SIR model, which is presented in Figure 3.1.

In this model, the population is broken down into three compartments: susceptibles \((S)\), infected \((I)\) and recovered \((R)\). Individuals move from the population
of susceptibles to the population of infected through the interactions between these populations at the rate of $\beta S\frac{I}{N}$, where $\beta$ represents the rate at which infected individuals give rise to new infections. Individuals then move from the infected compartment to the recovered compartment at the rate of $\alpha I$, where $\alpha$ represents the rate of recovery from the disease. The equations of the model are given as follows:

$$\frac{dS}{dt} = -\beta S\frac{I}{N}$$
$$\frac{dI}{dt} = \beta S\frac{I}{N} - \alpha I$$
$$\frac{dR}{dt} = \alpha I$$
$$N = S + I + R$$

We will now look at conditions which cause an epidemic to occur. For an epidemic to occur, the number of infected individuals needs to increase. Thus, an epidemic occurs if

$$\frac{dI}{dt} > 0$$
$$\beta S\frac{I}{N} - \alpha I > 0$$
$$\beta S\frac{I}{N} > \alpha I$$
$$\frac{\beta}{\alpha} > \frac{I}{N} \cdot \frac{N}{S}$$
$$\frac{\beta}{\alpha} > \frac{N}{S}$$
At the outset of an epidemic, the entire population is susceptible. Thus, \( S = N \) at \( t = 0 \), where \( t \) represents a time period. Substituting that in the inequality, we get \( \frac{\beta}{\alpha} > \frac{N}{N} = 1 \). Therefore, an epidemic will occur if \( \frac{\beta}{\alpha} > 1 \). \( \frac{\beta}{\alpha} \) is also known as the basic reproductive number or \( R_0 \). The basic reproductive number is a threshold that determines whether or not an outbreak of a disease will take place. It is often defined as the average number of infections generated by a "typical" infectious individual after its introduction in a population of susceptibles. Thus, an epidemic will occur if \( \frac{\beta}{\alpha} > 1 \) and will not occur if \( \frac{\beta}{\alpha} < 1 \).

We have thus, introduced the basic concepts of an epidemiological model. A more thorough mathematical presentation of the SIR model will be presented in Chapter 4. We can now move onto discussing the literature on the modeling of the prostitution and trafficking industry.

### 3.2 Study 1: Mathematical Modeling of the Sex Worker Industry as a Supply and Demand System

Davidoff et al (2006) [21] analyzes the commercial sex industry in the United States as a market of demand and supply. Here, women are assumed to represent the entire supply side of the market and men are assumed to represent the entire demand of the market. The authors assume that the male demand for commercial sex is the driving force behind this market and regardless of the amount of supply, the demand is always satisfied. Thus, the authors construct two separate models, one to represent supply and the other to represent demand, to understand the social dynamics of the market. They first analyze the models individually and finally proceed to explore the coupled system numerically.
3. A Review of Literature

The first model built by the authors is the ‘female model’ representing the supply side of the market and is presented in Figure 3.2.\(^1\)

![Diagram of the female supply model.](image)

The model consists of four distinct compartments: the susceptible class \((S)\), the commercial sex worker class \((P)\), the class consisting of commercial sex workers who are detained in jail \((J_f)\) and the rehabilitated class \((R)\). The ‘infected’ compartment in this model is assumed to be the population of commercial sex workers, \(P\). Prostitution is deemed to be the ‘disease’ in the system, and an active commercial sex worker is deemed to be an ‘infected individual.’

In the Figure 3.2, \(\mu_fN\) represents all individuals who are born into the system and are automatically assumed to enter the susceptible population \((S)\). The transfer of individuals from the susceptible to the infected population depends on the rate of contact between the two populations and a recruitment rate. This is given by \(\beta S \frac{P}{N}\),

\(^1\)Source: Davidoff et. al (2006)
where $\beta$ represents the per capita recruitment rate, $\frac{P}{N}$ represents the probability of coming into contact with a commercial sex worker and $S$ is the population of susceptibles. The authors assume extremely small fluctuations in the population size, making the birth rate equal the death rate, $\mu_f$. Thus, $\mu_f S$ represents the rate at which individuals naturally leave the population of susceptibles.

Once individuals enter the population of prostitutes or commercial sex workers ($P$), they are arrested and thus, enter the population of jailed workers ($J_f$), at a rate of $\gamma_f P$, where $\gamma_f$ represents the arrest rate of commercial sex workers. After serving their time in prison, the authors assume that a certain portion of the individuals resume being a commercial sex worker at the rate of $\delta_f \kappa_f J_f$, where $\delta_f$ represents the proportion of prostitutes who resume working and $\kappa_f$ represents the rate of leaving prison.

The proportion of individuals who choose to not resume working as a commercial sex workers ($(1 - \delta_f)$, can decide to enter rehabilitation programs at the rate of $(1 - \delta_f)\kappa_f J_f$, and thus transfer to the class of rehabilitated workers ($R$) or exit the system naturally at the rate of $\mu_f J$. Once in the rehabilitated class, individuals can either exit out of this compartment by returning to work as a prostitute due to a relapse caused by an interaction between a commercial sex worker and a rehabilitated sex workers. Thus, they move back into the population of commercial sex workers at a rate of $\rho R \frac{P}{N}$, where $\rho$ is the relapse rate of rehabilitated sex workers. The individuals who do not resume working exit the system naturally at the rate of $\mu_f R$.

Thus, the equations for the model are given as follows:
\[ \dot{S} = \mu_f N - \beta S \frac{P}{N} - \mu_f S \]
\[ \dot{P} = \beta S \frac{P}{N} - (\gamma_f + \mu_f)P + \delta_f \kappa_f J_f + \rho R \frac{P}{N} \]
\[ \dot{J}_f = \gamma_f P - (\kappa_f + \mu_f)J_f \]
\[ \dot{R} = (1 - \delta_f) \kappa_f J_f - \rho R \frac{P}{N} - \mu_f R \]

The second model built by the authors is the ‘male model’ representing the demand side of the prostitution market. A graphical representation of the transfer of individuals to and from compartments is given in Figure 3.3.²:

![Diagram of the male model](image)

**Figure 3.3:** Diagram of the male model.

As seen in the female model, this model also consists of four distinct compartments: the population of potential customers, \( C_p \), the population of abstainers, \( G \), the population of active customers, \( C_a \) and the population of the customers detained.

²Source: Davidoff et. al (2006)
in jail, \( J_m \). Prostitution is considered the ‘disease’ in this model as well, with an ‘infected individual’ representing an active customer of commercial sex. The model assumes a small, constant male population, \( M \), consisting of ‘sexually mature’ men from the age group 15-55. Since the population is constant, the birth rate equals the rate of naturally exiting the system due to maturing out of the age group, which is given by \( \mu_m \).

Men enter the model at a rate \( \Lambda \mu_m M \), where \( \Lambda \) represents the proportion of the entire male population which is mature enough to be considered a potential customer. A portion of potential customers, \( C_p \), are converted to active customers, \( C_a \), through interactions with existing active customers at a rate, \( \eta_1 C_p C_a M \), where \( \eta_1 \) represents recruitment rate from potential to active customers. Similarly, interactions with individuals from the population of abstainers, \( G \), may convert a portion of the potential customers to permanent abstainers. This is given by \( \eta_2 C_p G M \), where \( \eta_2 \) is the recruitment rate of potential customers to abstainers. Customers who are active, \( C_a \), are arrested at a \( \gamma_m C_a \), where \( \gamma_m \) represents the arrest rate of active customers. Due to an increase in arrests, some active customers choose to cease activity and thus, enter the population of potential customers at the rate, \( \omega J_m \), where \( \omega \) represents the rate of ceasing activity due to the knowledge of arrest. Finally, after serving their prison sentence, individuals may either resume consuming commercial sex at the rate, \( \delta_m \kappa_m J_m \), where \( \delta_m \) refers to the proportion of jailed customers that resume the purchase of commercial sex and \( \kappa_m \) refers to the rate of leaving jail, or they may choose to permanently abstain from the future consumption of commercial sexual services at a rate of \( (1 - \delta_m) \kappa_m J_m \).

The system of equations for the male model are given as follows:
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\[
\begin{align*}
\dot{C}_p &= \Lambda \mu m M - \eta_1 C_p \frac{C_a}{M} - \eta_2 C_p \frac{G}{M} + \omega C_a + (1 - \delta_m) \kappa_m J_m - \mu_mC_p \\
\dot{G} &= (1 - \Lambda) \mu m M + \eta_2 C_p \frac{G}{M} - \mu_m G \\
\dot{C}_a &= \eta_1 C_p \frac{C_a}{M} + \delta_m \kappa_m J_m - (\gamma_m + \omega + \mu_m) C_a \\
\dot{J}_m &= \gamma_m C_a - (\kappa_m + \mu_m) J_m
\end{align*}
\]

After creating the two models, the authors begin an individual qualitative analysis of each model and discuss the local stability of their equilibrium conditions. The authors begin by analyzing the male model and looking for the disease-free equilibrium, or a state without any active customers, which is referred to as the \(C_aFE\) (Active Customer Free Equilibrium) model. Thus, the following is obtained:

\[
C_aFE : (c^*_p, g^*, c^*_a, j^*_m) = (1 - g^*, g^*, 0, 0)
\]

To determine \(g^*\), the authors set the equations of the model to 0 and simultaneously solve for \(g^*\), obtaining

\[
g^* = \frac{1}{2} \left(1 - \frac{\mu}{\eta_2} + \sqrt{\left(1 - \frac{\mu^2}{\eta_2^2}\right) + 4 \frac{\mu}{\eta_2} (1 - \Lambda)}\right)
\]

The authors calculate the basic reproductive number, \(R_m\) using a variation of the next generator operator method that will be discussed in Chapter 4, to determine the stability of the disease-free equilibrium. The basic reproductive number is found to be:

\[
R_m = MD^{-1} = \left(\frac{\eta_1 c^*}{\omega + \gamma_m + \mu_m}\right) + \left(\frac{\delta_m \cdot \gamma_m}{\omega + \gamma_m + \mu_m} \cdot \frac{\kappa_m}{\mu_m + \kappa_m}\right)
\]
The first term in the equation above represents the average number of potential customers recruited per active customer during the length of time of them being active in the purchase of commercial sex. This is because $\eta_1 c^*$ represents the average number of potential customers recruited by a single active customer per unit of time and $\frac{1}{\omega + \gamma_m + \mu_m}$ represents the average length of time that a customer involved in the purchase of commercial sex is active. The second term in the equation represents the proportion of customers who resume buying commercial sex after completing their time in detainment. This is because $\frac{\gamma_m'}{\omega + \gamma_m + \mu_m}$ represents the fraction of active customers who got arrested, $\frac{\kappa_m}{\mu_m + \kappa_m}$ represents the proportion of detained customers who leave jail and $\delta_m$ represents the proportion of detained customers who resume visiting prostitutes after leaving jail. Thus, in this model, the basic reproductive number, $R_m$, represents the both the average number of customers recruited per active customer during their time of being active as well as the average number of detained customers who return to being active after completing their time in detention. The disease-free equilibrium or the $CaFE$ model, is stable as, $R_m$ is shown to be less than 1. However, since the $CaFE$ model depicts a situation which is not currently seen in U.S. cities, the authors proceed to look for conditions at which this model becomes unstable. Thus, a population of active customers is developed.

To observe the existence of this equilibrium, the authors plot the basic reproductive number against the active customer population and find that an active customer population is created when $R_m > 1$. This is shown in Figure 3.4 \(^3\).

---

\(^3\)Source: Davidoff et. al (2006)
After examining the stability of the male model, the authors move on to analyzing the stability conditions of the female model. Once again, they begin by looking at a disease-free equilibrium or a Prostitution Free Equilibrium (PFE). Thus, the following model is obtained:

$$\text{PFE} : (S^*, P^*, J^*, R^*) = (N, 0, 0, 0)$$

Since there is no prostitution in the system, all the individuals are in the susceptible class, $(S)$. To examine the effects of introducing a single commercial sex worker into the system, the authors once again use the next generator operator method described in Chapter 4 to find the Basic Reproductive Number to be:

$$R_f = \text{MD}^{-1} = \left( \frac{\beta}{\mu + \gamma_f} \right) + \left( \delta_y \cdot \frac{\gamma_y}{m \mu_f + \gamma_f} \cdot \frac{\kappa_f}{\mu_f + \kappa_f} \right)$$
In the expression above, the first term represents the average number of individuals recruited from the entire susceptible population (N=1) into the prostitution population by a single sex worker, divided by the average time spent in the prostitution class. The second term in the expression represents the individuals who once again enter the prostitution class after deciding to resume working after the completion of their jail term.

Looking at the Prostitution Prevalent Equilibrium (PPE) now, the authors observe that the system omits a backwards bifurcation when $R_f = 1$. Thus, there are many solutions to the system for when the populations approach the PPE. This is shown graphically in Figure 3.5.

Therefore, $R_f < 1$ does not imply stability of the PFE model. The authors argue that if we were to reduce $R_f$ below some threshold value, $R_c$, it would be possible to establish the trivial equilibrium. However as this is highly unlikely in the real

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4Source: Davidooff et. al (2006)
world as a large level of prostitution has existed due to the increasing demand of commercial sex, the authors emphasize that efforts need to be made in the demand side of the market if we were to reduce the level of infection, or prostitution.

To better analyze the changes in the model due to changes in parameter values, the authors perform sensitivity analysis on both the female and male reproductive numbers. In the male model, the authors find that $R_m$ is most sensitive to changes in the recruitment rate from potential to active customers, $\eta_1$, as well as the rate of ceasing activity due to an increase in the arrests of active customers, $\omega$. Since it is extremely difficult to control the recruitment of active customers, the authors argue that with efforts made to increase the arrests of active customers, it will be possible to reduce $R_m < 1$ such that the system approaches a prostitution-free equilibrium.

In the female model, the authors find that $R_f$ is most sensitive to changes in the recruitment rate, $\beta$ so much so that with small changes in $\beta$ lead to a significantly high changes in $R_f$. Thus, the different methods of recruitment into prostitution play a pivotal role in controlling the population of prostitutes.

The authors finally couple the individual systems and perform a numerical simulation of the threshold values to analyze the effect the on the infected population proportions, $c_a$ and $p$. In their coupled approach, some parameters in each model are made dependent on the populations in the other models. These effects are graphically shown in Figure 3.6.

As can be seen from Figure 3.6, there is not much change in the populations of both $C_a$ or $p$ when the threshold value of the male model is greater than 1, i.e. when $R_m > 1$. In that situation, no matter what the threshold value of $R_f$, a population of active customers, $c_a$ and active prostitutes, $p$ will always be established. When the $R_F < R_f^c$, we observe both the populations, $C_a$ and $p$ approach 0. However, like mentioned earlier, it is extremely impractical to try and reduce $R_f$ below a critical

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5Source: Davidoff et. al (2006)
threshold value in the real world. But if efforts are made to obtain a value for $R_m$, such that $R_m < 1$, both populations of $C_a$ and $p$ eventually approach the disease-free equilibrium. This is graphically shown in the lower right panel.

In conclusion, the authors argue that it is only through an increased government activity in the demand side of the market that the level of prostitution be controlled. Thus, the authors urge law-enforcement authorities to focus their efforts on male arrests rather than female arrests. As seen in this analysis, the dynamics of the male population are not very sensitive to the dynamics of the female population. However, the reverse is not true. Thus, by eliminating the demand from the male side, the system collapses. Although this study provides a good platform for the modeling of the prostitution industry, it is attributed with some major flaws, such as unreasonable parameter values producing exorbitantly high estimates of some
populations. We will build off of this model and analyze and discuss the impact of these parameters choices further in Chapters 5 and 6.

The next study extends the model presented by Davidoff et al (2006) [21] to solely analyze the supply side of prostitution.

### 3.3 Poverty and Prostitution in Nigeria

Oduwole and Shehu (2013) [37] incorporate the epidemiological model formulated by Davidoff et. al (2006) to obtain a non-violent approach to reduce the rate of prostitution in Nigeria. Davidoff et. al’s primary analysis dealt with analyzing the possible impacts of government influence on the demand side of the prostitution market to control the level of commercial sex workers. This study however, focuses on the impact of government efforts on the supply side of prostitution. Specifically, the study focuses on the impact of poverty on the dynamics of prostitution. According to the authors, poverty and prostitution are directly linked. Among the different pathways into prostitution that include child abuse, family breakdown, sexual abuse, etc., one of the greatest is poverty-induced circumstances, which constitutes the focal point of there analysis.

In this study, the authors assume a constant population, which is divided into five separate compartments. These compartments include, the non-impoverished or rich class, (N) the poverty class (P), the prostitution class (S), the disease infected class (I) and the rehabilitated class (R). A diagram of their model is given in Figure 3.7.  

Since the authors assumed a constant population in this model, the per capita birth rate, $\mu$, is equal to the per capita death rate, $\alpha$. Thus, we have $\mu = \alpha$. The rate of flow from the non-impoverished class to the poverty class is denoted by

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6Source: Oduwole and Shehu (2013)
$\beta$, and depends on both the unemployment as well as underemployment rate. $\omega$ denotes the rate at which individuals move from the poverty ($P$) class to the rehabilitation ($R$) class, due to the intervention of the government. The last term in the equation representing the poverty class, $\frac{\varphi PS}{T}$, denotes the transfer of individuals from the poverty class ($P$) to the prostitution class ($S$), based on the assumption that an individual enters the prostitution class only after coming into contact with another individual from the prostitution class, where $\varphi$ represents the rate at which individuals in the poverty class enter the prostitution class. Another method of entering the prostitution world, is through the rehabilitation class ($R$). This happens after an individual moves from the poverty ($P$) to prostitution ($S$) to rehabilitation class ($R$), and then enters the prostitution class ($S$) once again due to recidivism or relapse. This is represented by the term $\frac{\delta \varphi RS}{T}$, where $\delta$ represents the reduced rate at which individuals from the rehabilitation class ($R$) enter the prostitution class ($S$). The term $\eta$ represents the rate at which a prostitute gets infected with a sexually transmitted disease (STD) and the terms $\alpha_1$ and $\alpha_2$ represent the disease induced death rate in the infected class ($I$) and the rehabilitation class ($R$) respectively. Finally, $\phi$ represents the rate at which an infected prostitute gets transferred to the rehabilitation class ($R$) due to government intervention.
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The system of ordinary differential equations formulated in this study is given as follows:

\[
\frac{dN}{dt} = \mu T - (\mu + \beta)N \\
\frac{dP}{dt} = \beta N - (\mu + \omega)P - \frac{\varphi PS}{T} \\
\frac{dS}{dt} = \frac{\delta \varphi PS}{T} + \frac{\varphi RS}{T} - (\mu + \eta)S \\
\frac{dI}{dt} = \eta S - (\mu + \alpha_1 + \phi)I \\
\frac{dR}{dt} = \omega P + \phi I - (\mu + \alpha_2)R - \frac{\delta \varphi RS}{T} \\
N + P + S + I + R = T
\]

The authors now use Maple 15 to solve the system of equations numerically. Baseline parameters for Nigeria were obtained from the National Bureau of Statistics and the United States Agency for International Development. This data was used in conducting three experiments that dealt with measuring the effect of government intervention on the different population compartments. For this purpose, the parameters analyzed were the ones that affected recruitment into rehabilitation programs motivated by government efforts. These included the rate of which individuals move from the poverty class into the rehabilitation class, \( \omega \) and the rate at which infected prostitutes move into the rehabilitation class, \( \phi \). The values chosen for these parameters were given as follows:

Low values: \( \omega = \phi = 0.20 \)

High values: \( \omega = \phi = 0.75 \)
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The first experiment was performed to measure the impact on the poverty class \((P)\). The results graphically represented in Figure 3.8\(^7\).

![Figure 3.8: Effect of government intervention on the poverty class.](image)

As we can see from the Figure 3.8, a high rate of government intervention decreases the population of the poverty class, much faster than a low rate of government intervention. Thus, a low poverty class implies lesser individuals entering the prostitution class. Thus, the author emphasizes an increase in government action through programs such as soft loans, skill acquisition, etc.

The next experiment performed was to measure the impact of government intervention on the infected class \((I)\). This is graphically shown in Figure 3.9\(^8\). As seen from the graph, the authors find that high government intervention goes a long way in curbing the population of prostitutes who get infected. They emphasize the need for non-violent government programs such as free counseling, financial assistance, etc. in order to educate individuals on the consequences of prostitution, as well as rehabilitate them and promote self-employment.

\(^7\)Source: Oduwole and Shehu (2013)
\(^8\)Source: Oduwole and Shehu (2013)
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The final experiment performed by the authors was done to measure the impact of government efforts on the rehabilitated class ($R$). The results of their experiment are graphically represented in Figure 3.10.9.

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9Source: Oduwole and Shehu (2013)
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As seen in the Figure 3.10, we can observe that a high level of government intervention initially, increases the population of the rehabilitation class and eventually causes it to decrease due to a reduction in the poverty and infected classes. This shows the importance of rehabilitation programs and the need for government intervention to help reduce the population of prostitutes by decreasing rehabilitating and helping underprivileged individuals.

The primary conclusion from this study is that government intervention plays an important role in controlling the population of prostitutes in Nigeria. But the bottomline is that poverty eradication and control is the driving force behind a lower level of the population of prostitutes in the market, as is indicated by the results of their model. Although this study provides a informative insight into the dynamics and relationship between poverty and prostitution in Nigeria, the model used is highly simplistic and leaves out certain variables and considerations, such as NGO intervention, types of prostitutes involved, etc., that are important in fully understanding the workings of the market for prostitution. In Chapter 6, we will use Davidoff’s epidemiological model to analyze the market for prostitution and introduce the element of human trafficking to analyze the effect the reduction of the recruitment rate of trafficked victims (that is affected by poverty and other factors) has on the demand for prostitution.

We now discuss the final study which uses a regression model to look at the relationship between human trafficking and the legalization of prostitution.
3.4 **Does Legalized Prostitution Increase Human Trafficking?**

Cho et al. (2013) [21] studies the effects of the legal status of prostitution on the incidence of human trafficking inflows to countries by analyzing macroeconomic data in a regression model. The authors argue that theoretically, there are two contradictory effects of the legalization of prostitution on human trafficking inflows, a substitution effect away from trafficking and a scale effect increasing trafficking.

Starting with the demand side, the authors argue that the legalization of prostitution is hypothesized to raise the demand for prostitution, as some clients may have previously been deterred from consuming commercial sex services due to the risk of criminal prosecution as well as social and moral reasons. On the supply side, the authors say that legalizing prostitution will induce some potential sex workers to enter the market, namely those for whom the probability of being prosecuted was too high a risk to enter the market when prostitution was illegal. Thus, the supply of prostitution will increase as well, raising the equilibrium quantity of prostitution. This increase in equilibrium will raise the demand of trafficked sex workers and thus, will lead to an increase in human trafficking. On the other hand, some firms may want to take advantage of the legality of prostitution and recruit from local citizens who willingly want to enter this industry. Thus, this has a substitution effect away from trafficking. The net change in the incidence of human trafficking will depend on the magnitude of these effects. Since these two effects cancel each other out, the authors argue that economic theory alone is not sufficient to determine the effect that the legalization of prostitution would have on human trafficking.

Cho et al. (2013) [17] thus, turns to empirical analysis to analyze this issue. They take data from the report written by the United Nations Office on Drugs and Crime, which includes cross-country information on the reported incidence of
human trafficking in 161 countries. The dependent variable in this analysis captures
the incidence of human trafficking into a country while the independent variables
include explanatory variables such as GDP per capita, population, etc. This is
cumulatively given in a vector $X$. The other independent variable, Prostitution, is
a dummy variable which indicates whether or not prostitution is legal in a given
country. To mitigate country heterogeneity that may arise given the cross-sectional
nature of the data set, regional fixed effects are taken into account in the variable,
Region. The estimation regression equation of the research takes the following form:

$$y_i = \alpha + \beta_1 Prostitution + \beta_2 X_i + \beta_3 Region_1 + \epsilon_i$$

where,

$y_1$ = reported degree of human trafficking inflows in country $i$

$\alpha$ = quantity of trafficked victims

$X_i$ = vector of explanatory variables

Prostitution = dummy variable indicating whether or nor prostitution is legal

Region$_1$ = Regional fixed effects

$\epsilon_i$ = idiosyncratic error term

The net result (significant at the 5% level) from this empirical analysis indicates
that the scale effect dominates the substitution effect. Thus, the authors argue that
countries where prostitution is legalized experience a larger degree of human tra-
ficking inflows than countries with illegal prostitution. Thus, this study substantiates
our hypothesis that the legalization of prostitution leads to an eventual increase in
the level of human trafficking in a country.
3.5 Concluding Remarks

Both the studies by Davidoff et. al (2006) [21] and Oduwole and Shehu (2013) [37] focused on modeling the market for prostitution by using compartmental epidemiological models. Davidoff et. al (2006) created two separate models, an all-female supply model and an all-male demand model, and analyzed them both separately and together to study the dynamics of the prostitution market. Their main findings involved the importance of government efforts and legal activity on the demand side in order to control the population of prostitutes, as they found that the threshold values for the male model to be extremely sensitive to the rate of ceasing activity due to an increase in customer arrests. Another important finding was that the female supply system was extremely sensitive to the recruitment rate of prostitutes. A component of this recruitment rate, namely prostitution due to poverty-induced circumstances, was the focus of the analysis done by Oduwole and Shehu (2013). In their study, Oduwole and Shehu, focused only on the female, supply side of the prostitution market and showed that government induced poverty eradication and rehabilitation programs played a pivotal role in decreasing the population of prostitutes in Nigeria. We will delve deeper into the effects of this recruitment rate of prostitution when we build and simulate our trafficking model in Chapter 6. Cho et al. (2013) [17], on the other hand, moved away from microeconomic factors and used macroeconomic data to analyze the direct relationship between human trafficking and legalized prostitution. The major finding of the study was that trafficking is highest in countries where prostitution is legalized. This allowed us to further validated our hypothesis.

We have thus, briefly analyzed the market for prostitution through epidemiological models. In the next chapter, we will explore the fundamental concepts involved
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in epidemic theory in detail in order to establish the tools needed to build a system that can model human trafficking and prostitution as discussed in Chapter 6.
Epidemic Theory: A Mathematical Approach

In this chapter, we look at the mathematical detail of compartmental epidemiological models for disease transmission. We begin by providing a general overview of a simple SIR model and then proceed to explain the dynamics of the basic reproductive number and the analysis of the stability of the system. Finally, we conclude with an example of an SEIR model. This chapter is an exposition of Driessche and Watmough (2008). [48]

4.1 Basic SIR Model

Compartmental epidemiological models are mathematical models which analyze the establishment and spread of infectious diseases. In such models, the population is divided into different compartments depending on certain assumptions, with the infection or disease spread through the interaction of the agent, host and environment. The simplest form of an epidemiological model, is the basic SIR model in which populations are divided into three compartments based on the characterization of their state: susceptibles, infected and recovered. A diagram of the transmission of individuals from one compartment to another is given in Figure 4.1.
4. Epidemic Theory: A Mathematical Approach

In this model, $S$ represents the number of individuals who are susceptible to the disease, that is, those who have not yet been infected by it. $I$ represents the number of people who are infected with the disease and $R$ represents the number of people who have recovered from the disease. The total population in the model is given by $N$. A susceptible individual becomes infected by the contact between those infected and those susceptible. This is given by $\beta S \frac{I}{N}$, where $\beta$ represents the rate at which infected individuals give rise to new infections. An infected individual moves to the population of recovered individuals at the rate of $\alpha I$, where $\alpha$ represents the rate at which infected individuals recover from the disease. This model assumes a constant population with no vital dynamics (no birth and death rate). The only way a person can leave the susceptible group is to become infected and the only way a person can leave the infected group is to become recovered. After recovery, an individual is immune and no longer susceptible. The equations of the model are as follows:

$$\frac{dS}{dt} = -\beta S \frac{I}{N}$$
$$\frac{dI}{dt} = \beta S \frac{I}{N} - \alpha I$$
$$\frac{dR}{dt} = \alpha I$$

$$N = S + I + R$$

Given these assumptions, we now look at conditions which cause an epidemic
4. Epidemic Theory: A Mathematical Approach

to occur. For an epidemic to occur, the number of infected individuals needs to increase. Thus, an epidemic occurs if

\[
\frac{dI}{dt} > 0
\]

\[
\beta S \frac{I}{N} - \alpha I > 0
\]

\[
\beta \frac{S}{N} I > \alpha I
\]

\[
\frac{\beta}{\alpha} > \frac{I}{S} \cdot \frac{N}{S}
\]

\[
\frac{\beta}{\alpha} > \frac{N}{S}
\]

Since we know that at the outset of an epidemic, the entire population is susceptible, \( S = N \) at \( t = 0 \), where \( t \) represents a time period. Substituting that in the inequality, we get \( \frac{\beta}{\alpha} > \frac{N}{N} = 1 \). Therefore, an epidemic will occur if \( \frac{\beta}{\alpha} > 1 \). \( \frac{\beta}{\alpha} \) is also known as the basic reproductive number or \( R_0 \). The basic reproductive number is a threshold that determines whether or not an outbreak of a disease will take place. Thus, an epidemic will occur if \( \frac{\beta}{\alpha} > 1 \) and will not occur if \( \frac{\beta}{\alpha} < 1 \). The basic reproductive number is one of the most important parameters in epidemic theory and is central to most studies of the dynamics of infectious diseases. \( R_0 \) is most often defined as the average number of infections generated by a “typical” infectious individual after its introduction in a population of susceptibles. The calculation of the basic reproductive number may be much more complicated in more structured models than the simple one we have looked at in this example, but the essential concept of the basic reproductive number does not change. The following section provides an alternate method for finding the basic reproductive number that applies to more complex models.
4. Epidemic Theory: A Mathematical Approach

4.2 Next Generator Operator Method

In this section, we look at a more generalized method of calculating the basic reproductive number using the next generator operator method. Let us suppose that there are \( n \) disease compartments (such as active disease, remission carriers, etc.) and \( m \) non-disease compartments (such as susceptibles, recovered etc.). A compartment is called a ‘disease compartment’ if the individuals in that compartment are infected. Let \( x \in \mathbb{R}^n = (x_1, x_2, x_3, ..., x_n) \) and \( y \in \mathbb{R}^m = (y_1, y_2, y_3, ..., y_m) \) be the subpopulations of these compartments. We will divide the change in the population of disease compartments into an increase component and a decrease component. Further, let \( F_i \) denote the rate secondary infections increase the population of the \( i \)th disease compartment and \( V_i \) denote the rate disease progression, death and recovery decrease the population of the \( i \)th disease compartment. The non-disease compartment rate of changes are not broken down and simply represented by the functions, \( g_j \). Thus, the model can be written as follows:

\[
x'_i = F_i(x, y) - V_i(x, y), \quad i = 1, ..., n \tag{4.1}
\]

\[
y'_j = g_j(x, y), \quad j = 1, ..., m \tag{4.2}
\]

where \( x'_i \) represents the changes in the disease compartments and \( y'_j \) represents the changes in the non-disease compartments. For example, in our SIR model, since we only have one disease compartment (I), we would have the following:

\[
F = \beta S \frac{I}{N} \quad \text{and} \quad V = \alpha I
\]

\[
x' = F_i(x, y) - V_i(x, y)
\]

\[
= \beta S \frac{I}{N} - \alpha I
\]
4. Epidemic Theory: A Mathematical Approach

We now look at the assumptions involved in the model that establish a state of equilibrium.

4.2.1 Assumptions

The following assumptions are made in order to ensure the existence of equilibria:

1. $\mathcal{F}_i(0, y) = 0$ and $\mathcal{V}_i(0, y) = 0$ for all $y \geq 0$ and $i = 1, ..., n$. All new infections are secondary infections that arise only from infected hosts. If there are no infected hosts, there will be no immigration of hosts from non-disease compartments into disease compartments. This assumption also ensures that a disease-free set $(0, y)$, or a set with no new infected individuals ($x = 0$), is invariant. Thus, a solution with no infected hosts at any given point in time will be free of that infection for the rest of time. Therefore, the disease-free equilibrium, or the equilibrium for the set for which there are no new infected individuals, is also the equilibrium of the full system. For example, in the SIR model, if $\beta S_I N = 0$, implying that there are no new infected individuals, then the solution of this system at the current time $t$, will be free of the infection for the rest of time.

2. $\mathcal{F}_i(x, y) > 0$ for all non-negative $x$ and $y$ and $i = 1, ..., n$. Since $\mathcal{F}$ represents the rate at which new infections increase the $i$th disease compartment, it cannot be negative.

3. $\mathcal{V}_i(x, y) \leq 0$ whenever $x_i = 0$ for $i = 1, ..., n$. Since $\mathcal{V}_i$ represents the net outflow of compartment $i$, it must be negative, thus representing only an inflow, whenever compartment $i$ is empty.

4. $\sum_{i=1}^{n} \mathcal{V}_i(x, y) \geq 0$ for all non-negative $x$ and $y$. This sum represents the total net outflow from all the infected compartments. Every inflow for a $\mathcal{V}_i$ is an outflow for another $\mathcal{V}_i$. Thus, in the end, the total sum reflects only the
outflows. Terms in the model which lead to an increase in \( \sum_{i=1}^{n} x_i \) are assumed to be new infections and thus belong in \( \mathcal{F} \).

5. The disease-free system, \( y' = g(0, y) \), has a unique asymptotically stable equilibrium. Thus, all the solutions with initial conditions of the form \((0, y)\) approach a fixed point \((0, y_0)\) as \( t \to \infty \). This point is known as the disease-free equilibrium. Thus, any solution for a set with no new infected individuals initially, will be the solution to the entire system.

4.2.2 Linearization

Since compartmental epidemiological models are inherently nonlinear models which are often very complex, it is difficult to analyze their behaviour. Thus, we look for approximations to these nonlinear systems around an equilibrium point, to be able to study the behaviour of the nonlinear system in a neighborhood of such a point. In this section, we will look at the linearization of the disease compartments around the disease-free equilibrium \((0, y_0)\), to determine the initial ability of a disease to spread through the system. Since our current model is a generalized model with an indefinite amount of disease and non-disease compartments, we will look at an example of the linearization of a simpler model with fewer compartments and then extend our analysis to a more generalized setting. Note that the basic SIR model is just a further simplification of the example presented below.

Let us look at the linearization of an epidemiological model with 3 disease \((n = 3)\) and 2 non-disease \((m = 2)\) compartments. Let \( \vec{x} \) represent the vector containing all the disease compartments and \( \vec{y} \) represent the vector containing all other compartments. Thus, we have:

\[
\vec{x} = \langle x_1, x_2, x_3 \rangle
\]
The linearization of a function of two variables is given as follows:

\[L(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)\]

The linearization of functions of many variables, \((S\text{ in the SIR model})\), just expand similarly. We do a linearization for the right hand side of each equation in the model. Thus for our example, the linearization for \(x'_1\), where \(f = F_i - V_i\), is as follows:

\[
L(x', y) = \left[ F_1(x^0_0, y^0_0) - V_1(x^0_0, y^0_0) \right] + \left[ \frac{\partial(F_i - V_1)}{\partial x_i}(x^0_0, y^0_0)(x_1 - x_0) \right] + \left[ \frac{\partial(F_i - V_1)}{\partial y_1}(x^0_0, y^0_0)(y_1 - y_0) \right] \\
+ \left[ \frac{\partial(F_i - V_1)}{\partial x_2}(x^0_0, y^0_0)(x_2 - x_0) \right] + \left[ \frac{\partial(F_i - V_1)}{\partial x_3}(x^0_0, y^0_0)(x_3 - x_0) \right] \\
+ \left[ \frac{\partial(F_i - V_1)}{\partial y_2}(x^0_0, y^0_0)(y_2 - y_0) \right]
\]

\[
L(x', y) = \left[ F_1(x^0_0, y^0_0) - V_1(x^0_0, y^0_0) \right] + \left[ \nabla_x (F_i - V_1)(x^0_0, y^0_0) \cdot (x' - x_0) \right] \\
+ \left[ \nabla_y (F_i - V_1)(x^0_0, y^0_0) \cdot (y' - y_0) \right]
\]  \hspace{1cm} (4.3)

Similarly, the linearization of \(x'_2\) and \(x'_3\) gives us the following:

\[
x'_2 : \left[ F_2(x^0_0, y^0_0) - V_2(x^0_0, y^0_0) \right] + \left[ \nabla_x (F_2 - V_2)(x^0_0, y^0_0) \cdot (x' - x_0) \right] + \left[ \nabla_y (F_2 - V_2)(x^0_0, y^0_0) \cdot (y' - y_0) \right]
\]  \hspace{1cm} (4.4)
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\[ x_3' = \left[ F_3(x_0', y_0') - V_3(x_0', y_0') \right] + \left[ \nabla_x (F_3 - V_3)(x_0', y_0') \cdot (x' - x_0) \right] + \left[ \nabla_y (F_3 - V_3)(x_0', y_0') \cdot (y' - y_0) \right] \]

(4.5)

Since the disease-free equilibrium implies that there are no infected hosts, we have the disease free equilibrium as \((x_0', y_0') = (0, 0)\), where:

\[ x_0' = < x_{0,1}, x_{0,2}, x_{0,3} > = < 0, 0, 0 > \]
\[ y_0' = < y_{0,1}, y_{0,2} > \]

Assumption 1 tells us that since we have no infected hosts, there is no immigration of hosts from non-disease compartments to disease compartments. Thus, we have \( F_i(0, y) = 0 \) and \( V_i(0, y) = 0 \) for all \( y \geq 0 \ i = 1, ..., n \). Therefore, the first component of our linearized equations (4.3 - 4.5) is now:

\[ F_i(x_0', y_0') - V_i(x_0', y_0') = F_i(0, y_0') - V_i(0, y_0') = 0, \quad \text{where } i = 1, 2 \text{ and } 3. \]

Looking at the last component of our linearized equations (4.3 - 4.5), since \( (F_i - V_i) \) represents the change in the disease compartments whereas \( y_j \) represents the subpopulations of the non-disease compartments, we have:

\[ \frac{\partial (F_i - V_i)}{\partial y_j} (x_0', y_0') = \frac{\partial (F_i - V_i)}{\partial y_j} (0, y_0') = \lim_{h \to 0} \frac{(F_i - V_i)(0, y_{0,j} + h) - (F_i - V_i)(0, y_{0,j})}{h} = 0 \]

where \( i = 1, 2, 3 \) and \( j = 1, 2 \).

Thus, we have:

\[ \nabla_y (F_1 - V_1)(x_0', y_0') \cdot (y' - y_0') = 0 \cdot (y' - y_0') = 0 \]
Substituting in 0 for the first and last components of our linearized equations (4.3 - 4.5), we now obtain the following:

\[
L(\tilde{x}, \tilde{y}) = \left[ \mathcal{F}_1(x_0, y_0) - \mathcal{V}_1(x_0, y_0) \right] + \left[ \nabla_x(\mathcal{F}_1 - \mathcal{V}_1)(x_0, y_0) \cdot (\tilde{x} - x_0) \right] + \left[ \nabla_y(\mathcal{F}_1 - \mathcal{V}_1)(x_0, y_0) \cdot (\tilde{y} - y_0) \right] = 0 + \left[ \nabla_x(\mathcal{F}_i - \mathcal{V}_i)(x_0, y_0) \cdot (\tilde{x} - x_0) \right] + 0 = \nabla_x(\mathcal{F}_i - \mathcal{V}_i)(x_0, y_0) \cdot (\tilde{x} - x_0) = (M - D) \cdot (\tilde{x} - x_0) = (M - D) \cdot (\tilde{x} - 0) = (M - D) \cdot \tilde{x}
\]

where \(M\) and \(D\) are the \(n \times n\) matrices given by

\[
M = \begin{bmatrix}
\nabla_x \mathcal{F}_1(x_0, y_0) \\
\nabla_x \mathcal{F}_2(x_0, y_0) \\
\nabla_x \mathcal{F}_3(x_0, y_0)
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial \mathcal{F}_1}{\partial x_1}(0, y_0) & \frac{\partial \mathcal{F}_1}{\partial x_2}(0, y_0) & \frac{\partial \mathcal{F}_1}{\partial x_3}(0, y_0) \\
\frac{\partial \mathcal{F}_2}{\partial x_1}(0, y_0) & \frac{\partial \mathcal{F}_2}{\partial x_2}(0, y_0) & \frac{\partial \mathcal{F}_2}{\partial x_3}(0, y_0) \\
\frac{\partial \mathcal{F}_3}{\partial x_1}(0, y_0) & \frac{\partial \mathcal{F}_3}{\partial x_2}(0, y_0) & \frac{\partial \mathcal{F}_3}{\partial x_3}(0, y_0)
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
\nabla_x \mathcal{V}_1(x_0, y_0) \\
\nabla_x \mathcal{V}_2(x_0, y_0) \\
\nabla_x \mathcal{V}_3(x_0, y_0)
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial \mathcal{V}_1}{\partial x_1}(0, y_0) & \frac{\partial \mathcal{V}_1}{\partial x_2}(0, y_0) & \frac{\partial \mathcal{V}_1}{\partial x_3}(0, y_0) \\
\frac{\partial \mathcal{V}_2}{\partial x_1}(0, y_0) & \frac{\partial \mathcal{V}_2}{\partial x_2}(0, y_0) & \frac{\partial \mathcal{V}_2}{\partial x_3}(0, y_0) \\
\frac{\partial \mathcal{V}_3}{\partial x_1}(0, y_0) & \frac{\partial \mathcal{V}_3}{\partial x_2}(0, y_0) & \frac{\partial \mathcal{V}_3}{\partial x_3}(0, y_0)
\end{bmatrix}
\]

Thus, extending the linearization of the disease-free components to a generalized setting, if we have \(n\) disease compartments and \(m\) non-disease compartments with \(x \in \mathbb{R}^n\) and \(y \in \mathbb{R}^m\), then the linearization of \(x'\) is given by

\[
x' = (M - D)\tilde{x},
\]
where $M$ and $D$ are $n \times n$ matrices with the $(i,k)$ entry as follows:

\[
M = \frac{\partial F_i}{\partial x_k}(\vec{0}, \vec{y}_0) \quad \text{and} \quad D = \frac{\partial V_i}{\partial x_k}(\vec{0}, \vec{y}_0)
\] (4.6)

where $i, k = 1, 2, ..., n$.

We will show that due to Assumption 5, the stability of the original system (4.1 - 4.2), is determined through the stability of the linear system given by (4.6). The proof of this is presented in the next section.

4.2.3 Proof of the Stability of the System

In this section, we prove that the stability of the linearized system yields the stability of the nonlinear epidemiological model. Before delving into the proof, we first identify some key concepts that will later be used in proving this claim.

Let us define a nonnegative matrix $A$, as a matrix with elements, $a_{i,j} \geq 0$ for all $i, j$. In this case, we write $A \geq 0$. If a matrix $T$, is of the form $T = sI - B$, where $I$ is the identity matrix, $B \geq 0$ and $s > 0$, it is said to have a Z-sign pattern. In other words, $t_{i,j} \leq 0$ when $i \neq j$ (the off-diagonal entries of the matrix are nonpositive). An example of a matrix with the Z-sign pattern is as follows:

\[
T = \begin{bmatrix}
0 & -8 & -17 \\
-3 & 1 & -9 \\
-24 & -5 & 2
\end{bmatrix}
\]

For this matrix, a possible form of $sI - B$, where $s = 2$ and $B = \begin{bmatrix} 2 & 8 & 17 \\ 3 & 1 & 9 \\ 24 & 5 & 0 \end{bmatrix}$, is given as follows:
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\[ sI - B = s \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 8 & 17 \\ 3 & 1 & 9 \\ 24 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -8 & -17 \\ -3 & 1 & -9 \\ -24 & -5 & 2 \end{bmatrix} = T \]

In addition, if the spectral radius\(^1\) of \( B \) is less than \( s \), i.e. if \( \rho(B) \leq s \), then \( B \) is called an \textbf{M-matrix}. [15] Before proceeding to the proof of the local stability of a nonlinear system of epidemiological equations, we discuss two theorems and three lemmas relating to M-matrices. Theorems 1 and 2, as well as Lemma 1 are not proven, but simply just cited, as they are matrix theory results taken from [15], whereas Lemmas 2 and 3 as well as Theorem 3 are proven, as they are results directly from the work in this paper.

\textbf{Theorem 1.} Let \( A \) be an \( n \times n \) real matrix with \( a_{i,j} \leq 0 \) for all \( i \neq j \). \( A \) is a non-singular \textbf{M-matrix} if and only if \( A \) has all positive diagonal elements and there exists a positive

\(^1\)The spectral radius, \( \rho(A) \), of a matrix \( A \), is the maximum of the moduli of the eigenvalues of \( A \). [48]

\(^2\)A discussion of the spectral radius is beyond the scope of this thesis, but we will use the concept as part of the theory that is presented.
diagonal matrix $D$, such that $AD$ is strictly diagonally dominant$^3$; i.e.

$$a_{ii}d_i > \sum_{j \neq i} |a_{ij}|d_j, \quad i = 1, \ldots, n \quad [15]$$

Theorem 2. Let $A$ be an $n \times n$ real matrix with $a_{i,j} \leq 0$ for all $i \neq j$. $A$ is a non-singular $M$-matrix if and only if every real eigenvalue of $A$ is positive. [15]

Lemma 1. If $A$ has the Z-sign pattern, then $A$ is inverse-positive; that is, $A^{-1}$ exists and $A^{-1} \geq 0$ if and only if $A$ is a non-singular $M$-matrix. [15]

Lemma 2. Let $M$ and $D$ be as defined in (4.6) of Section 4.2.2, i.e.

$$M = \frac{\partial F_i}{\partial x_k}(\vec{0}, \vec{y}_0) \quad \text{and} \quad D = \frac{\partial V_i}{\partial x_k}(\vec{0}, \vec{y}_0)$$

where $i, k = 1, 2, \ldots, n$. Then, $MD^{-1}$ is nonnegative.

Proof. Let $M = \frac{\partial F_i}{\partial x_k}(\vec{0}, \vec{y}_0) = \lim_{h \to 0} \frac{F_i(0 + h, \vec{y}_0) - F_i(0, \vec{y}_0)}{h}$. From Assumption 1 we have $F_i(0, \vec{y}_0) = 0$ and from Assumption 2 we have $F_i(0 + h, \vec{y}_0) \geq 0$. Thus, each entry of the matrix $M$ is nonnegative. As such, $M \geq 0$. Similarly $D = \frac{\partial V_i}{\partial x_k}(\vec{0}, \vec{y}_0) = \lim_{h \to 0} \frac{V_i(0 + h, \vec{y}_0) - V_i(0, \vec{y}_0)}{h}$. We know from Assumption 1 that $V_i(0, y_0) = 0$ and from Assumption 3 that $V_i(0 + h, y_0) \leq 0$. Thus, the off-diagonal entries of the matrix $D$ are negative or zero. Thus, $D$ has the Z-sign pattern. The first column sum of $D$ has the following form:

$^3$ A matrix is strictly diagonally dominant if the diagonal entry in every row or column is greater than the sum of all the other entries in that row or column.

$^4$ Recall that $F_i$ denotes the rate secondary infections increase the population of the $i$th disease compartment and $V_i$ denotes the rate disease progression, death and recovery decrease the population of the $i$th disease compartment.
\[ \frac{\partial V_1}{\partial x_1} + \frac{\partial V_2}{\partial x_1} + \frac{\partial V_3}{\partial x_1} + \ldots + \frac{\partial V_n}{\partial x_1} \]

\[ = \lim_{h \to 0} \frac{V_1(0 \pm h, 0_2, 0_3, \ldots, 0_k, y_0) - V_1(\vec{y}_0) + \ldots + V_i(0 \pm h, 0_2, 0_3, \ldots, 0_k, y_0) - V_i(\vec{y}_0)}{h} \]

\[ = \lim_{h \to 0} \frac{\sum_{i=1}^n V_i(0_1 \pm h, 0_2, 0_3, \ldots, 0_k, y_0)}{h} \quad \text{(Using Assumption 1)} \]

\[ \geq 0 \quad \text{(Using Assumption 4)} \]

Similarly, we have that all column sums of \( D \) are nonnegative. Since the off-diagonal entries of \( D \) are negative or 0 and the column sums are positive or 0, it must be that the diagonal entry in each column is positive. Thus, \( D \) is diagonally dominant. Therefore, by Theorem 1, \( D \) is a (possibly singular)\(^5\) M-matrix. For our analysis, it is assumed that \( D \) is nonsingular. In this case, using Lemma 1, we know that \( D^{-1} \geq 0 \). Therefore, \( MD^{-1} \geq 0 \).

**Lemma 3.** If \( M \) is nonnegative and \( D \) is a nonsingular M-matrix, then \( R_0 = \rho(MD^{-1}) < 1 \) if and only if all eigenvalues of \((M - D)\) have negative real parts. \([48]\)

**Proof.** Let \( M \geq 0 \) and \( D \) be a nonsingular M-matrix. Using Lemma 1 and Lemma 2, we know that \( MD^{-1} \geq 0 \). Thus, \((I - MD^{-1})\) has the Z-sign pattern. Using Lemma 1, we know that \((I - MD^{-1})^{-1} \geq 0 \) if and only if \( \rho(MD^{-1}) < 1 \). Now, consider the equalities \((D - M)^{-1} = (D - MD^{-1}D)^{-1} = [(I - MD^{-1})D]^{-1} = D^{-1}(I - MD^{-1})^{-1} \) and \( D(D - M)^{-1} = I + M(D - M)^{-1} \). Due to these equalities, we know that \((D - M)^{-1} \geq 0 \) if and only if \((I - MD^{-1})^{-1} \geq 0 \). Since \((D - M)\) has the Z-sign pattern, using Lemma 1 we know that \((D - M)^{-1} \geq 0 \) if and only if \((D - M)\) is a nonsingular M-matrix. Using Theorem 2 we also know that the eigenvalues of \((D - M)\) have positive real

\(^5\)If all the entries in the column are zero, this could give us \( \det(A) = 0 \). Thus, the matrix may not be nonsingular.
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parts. Therefore, this implies that the eigenvalues of \((M - D)\) have negative real parts. Thus, the claim holds. □

We can now move on to proving the stability of the system as we have developed the tools needed for this proof.

**Theorem 3.** Consider the disease transmission model, as defined in (4.1 - 4.2) of section 4.2, given as follows:

\[
\frac{dx_i}{dt} = F_i(x, y) - V_i(x, y), \quad i = 1, \ldots, n \\
\frac{dy_j}{dt} = g_j(x, y), \quad j = 1, \ldots, m
\]

where \(x_i\) represents the changes in the disease compartments and \(y_j\) represents the changes in the non-disease compartments. The disease-free equilibrium of the system is locally asymptotically stable if \(R_0 = \rho(MD^{-1}) < 1\), but unstable if \(R_0 > 1\).

**Proof.** Let \(M\) and \(D\) be as defined in (4.6) of section 4.2.2 and let \(J_{21}\) and \(J_{22}\) be the matrices of the partial derivatives of \(g\) with respect to \(x\) and \(y\) evaluated at the disease-free equilibrium.

The Jacobian matrix for the linearization of the system evaluated at the disease-free equilibrium has the following block structure: [48]

\[
T = \begin{bmatrix}
M - D & 0 \\
J_{21} & J_{22}
\end{bmatrix}
\]

where, \(M - D = \)

\[
\begin{bmatrix}
\frac{\partial F_1}{\partial x_1}(\vec{0}, \vec{y_0}) - \frac{\partial V_1}{\partial x_1}(\vec{0}, \vec{y_0}) & \frac{\partial F_1}{\partial x_2}(\vec{0}, \vec{y_0}) - \frac{\partial V_1}{\partial x_2}(\vec{0}, \vec{y_0}) & \frac{\partial F_1}{\partial x_3}(\vec{0}, \vec{y_0}) - \frac{\partial V_1}{\partial x_3}(\vec{0}, \vec{y_0}) \\
\frac{\partial F_2}{\partial x_1}(\vec{0}, \vec{y_0}) - \frac{\partial V_2}{\partial x_1}(\vec{0}, \vec{y_0}) & \frac{\partial F_2}{\partial x_2}(\vec{0}, \vec{y_0}) - \frac{\partial V_2}{\partial x_2}(\vec{0}, \vec{y_0}) & \frac{\partial F_2}{\partial x_3}(\vec{0}, \vec{y_0}) - \frac{\partial V_2}{\partial x_3}(\vec{0}, \vec{y_0}) \\
\frac{\partial F_3}{\partial x_1}(\vec{0}, \vec{y_0}) - \frac{\partial V_3}{\partial x_1}(\vec{0}, \vec{y_0}) & \frac{\partial F_3}{\partial x_2}(\vec{0}, \vec{y_0}) - \frac{\partial V_3}{\partial x_2}(\vec{0}, \vec{y_0}) & \frac{\partial F_3}{\partial x_3}(\vec{0}, \vec{y_0}) - \frac{\partial V_3}{\partial x_3}(\vec{0}, \vec{y_0})
\end{bmatrix}
\]
The general solution to a system of two dimensional linear differential equations takes the form: $c_1 e^{\lambda_1 t} v_1 + c_2 e^{\lambda_2 t} v_2$, where $c_1$ and $c_2$ are arbitrary constants, $\lambda_1$ and $\lambda_2$ are eigenvalues and $v_1$ and $v_2$ are eigenvectors corresponding to those eigenvalues.

Now, if we consider $\lambda = a + ib$, using Euler’s formula, we get,

$$e^{\lambda t} = e^{(a+ib)t} = e^{at}(\cos bt + i \sin bt)$$

Thus, if $a > 0$, the solutions unboundedly increase at the rate of $e^{at}$ and if $a < 0$, the solutions decrease to 0. Since the behaviour of non-linear systems can be locally approximated by linear systems, for a system to be locally asymptotically stable, the real parts of the eigenvalues of the Jacobian need to be negative. Since the Jacobian is a lower triangular matrix, we know that its eigenvalues are $(M - D)$ and $J_{22}$ [40]. Since the eigenvalues of $J_{22}$ have negative real parts using Assumption 5, we see that the disease-free equilibrium is locally asymptotically stable if the eigenvalues of $(M - D)$ have negative real parts. Since $M$ is nonnegative and $D$ is a nonsingular M-matrix, by Lemma 3, all the eigenvalues of $(M - D)$ have negative real parts if and only if $\rho(MD^{-1}) < 1$. Thus, the system is locally asymptotically stable if $R_0 < 1$. 

$$J_{21} = \begin{bmatrix}
\frac{\partial g_1}{\partial x_1}(\vec{0}, \vec{y}_0) & \frac{\partial g_1}{\partial x_2}(\vec{0}, \vec{y}_0) & \frac{\partial g_1}{\partial x_3}(\vec{0}, \vec{y}_0) \\
\frac{\partial g_2}{\partial x_1}(\vec{0}, \vec{y}_0) & \frac{\partial g_2}{\partial x_2}(\vec{0}, \vec{y}_0) & \frac{\partial g_2}{\partial x_3}(\vec{0}, \vec{y}_0)
\end{bmatrix}$$

$$J_{22} = \begin{bmatrix}
\frac{\partial g_1}{\partial y_1}(\vec{0}, \vec{y}_0) & \frac{\partial g_1}{\partial y_2}(\vec{0}, \vec{y}_0) \\
\frac{\partial g_2}{\partial y_1}(\vec{0}, \vec{y}_0) & \frac{\partial g_2}{\partial y_2}(\vec{0}, \vec{y}_0)
\end{bmatrix}$$
To prove the instability condition of the system, we alter $D$ by $\epsilon$ and use a continuity argument to prove the claim. Let $R_0 \leq 1$. Then for any $\epsilon > 0$, we have that $((1 + \epsilon)I - MD^{-1})$ is a nonsingular M-matrix. Therefore by Lemma 1, we know that its inverse must be greater than 0, i.e. $((1 + \epsilon)I - MD^{-1})^{-1} \geq 0$. Now since $M$ is nonnegative and $D$ is a nonsingular M-matrix, using Lemma 3, we know that for any $\epsilon$, it must be that all the eigenvalues of $((1 + \epsilon)D - M)$ have positive real parts. Taking the limit of $\epsilon$ to 0, it follows that the eigenvalues of $(D - M)$ have nonnegative real parts. Thus, if $R_0 > 1$, some eigenvalue of $(D - M)$ has a negative real part. In other words, we know that there exists some eigenvalue of $(M - D)$ has a positive real part. Therefore, we can conclude that the system is unstable.

We will now prove the reverse side of the equality. Assume that the eigenvalues of $(D - M)$ have nonnegative real parts. Thus, for any $\epsilon > 0$, $(D + \epsilon I - M)$ is a nonsingular M-matrix and we know by the proof of Lemma 3 that $\rho(M(D + \epsilon I)^{-1}) < 1$. Since $\epsilon$ is arbitrary, taking the limit of $\epsilon$ to 0, we know that $\rho(MD^{-1}) \leq 1$. Thus, there exists some eigenvalue of $(D - M)$ which has a negative real part if and only if $\rho(MD^{-1}) > 1$. In other words, we know that there exists some eigenvalue of $(M - D)$ has a positive real part. Thus, the system is unstable. \[48\]

In this section, we have proved the stability of an epidemiological system. In section 4.2.4, we will further delve into the concept of the basic reproductive number given by $R_0 = \rho(MD^{-1})$.

### 4.2.4 Basic Reproductive Number

In this section, we will explore the concepts associated with the basic reproductive number. The number of infections produced by a single infectious individual after being introduced in a population of susceptibles, or the basic reproductive number, is often expressed as the product of the expected duration of an infectious period and the rate at which infections occur. For example, if the infectious period for a
disease is approximately 10 days, and the rate of occurrence is 0.02 infections/day, then the basic reproductive number, $R_0$, is given by $10 \times 0.2 = 2$ infections. We begin by first discussing the expected duration of an infectious period. For our generalized model, the expected time spent by an infected individual in each compartment is given by the following integral:

$$\int_0^\infty \phi(t, x_0) dt$$

where $\phi(t, x_0)$ represents the solution to $\ddot{x} = (M - D) \cdot \dot{x}$. The $ith$ component of this solution can be interpreted as the probability that an individual is in the disease state $i$ at time $t$. Thus, by taking the integral of this solution, we obtain the expected duration of the infectious period. We assume that there is no inflow of new infections, thus $(M = 0)$, and a nonnegative initial condition, which implies that although there are no new infections, there exists an initial infected population, $x_0$. Given these constraints, we have:

$$\ddot{x} = (M - D) \cdot \dot{x} = (0 - D) \cdot \dot{x} = -D \dot{x}, \quad \dot{x}(0) = x_0$$

We now discuss $\phi(t, x_0)$, the solution to $\ddot{x} = -D \dot{x}$, in order to be able to discuss the solution to its integral, $\int_0^\infty \phi(t, x_0) dt$. The solution to $\ddot{x} = -D \dot{x}$ is given as follows:

$$\frac{d\dot{x}}{dt} = -D \dot{x}$$

$$\int \frac{d\dot{x}}{\dot{x}} = \int -D \ dt$$

$$ln(\dot{x}) = -Dt + c$$

$$\dot{x} = e^{-Dt + c}$$

$$\dot{x} = e^{-Dt} \cdot e^c$$
When \( t = 0 \), we get \( \vec{x} = e^c \). Since \( \vec{x}(t) = \vec{x}(0) = x_0 \), we have \( \vec{x} = e^{-Dt} \cdot e^c = e^{-Dt} \cdot x_0 \).

Since \( D \) is a matrix, we need to define \( e^D \). This is done by a Taylor series expansion. The Taylor series expansion for the function, \( e^t \) is given as follows:

\[
e^t = \sum_{n=0}^{\infty} \frac{t^n}{n!}
\]

For a matrix, the Taylor series expansion is altered to: [27]

\[
e^D = \sum_{n=0}^{\infty} \frac{D^n}{n!}
\]

Therefore, \( e^D = I + D + \frac{D^2}{2!} + \frac{D^3}{3!} + \frac{D^4}{4!} + \ldots + \frac{D^k}{k!} \). The convergence of the series for all \( t \) is given as follows:

\[
\frac{d}{dt} e^{Dt} = \sum_{n=0}^{\infty} n \frac{t^{n-1} D^n}{n!} = \sum_{n=1}^{\infty} \frac{t^{n-1} D^n}{(n-1)!} = D \sum_{n=1}^{\infty} \frac{t^{n-1} D^{n-1}}{(n-1)!} = D \sum_{m=0}^{\infty} \frac{t^m D^m}{m!} = De^{Dt}.\] [27]

Thus, we know that the series converges for all \( t \). We now have, \( \int_0^\infty \phi(t, \vec{x}_0) \ dt = \int_0^\infty e^{-Dt} \cdot \vec{x}_0^\prime \ dt = D^{-1} \vec{x}_0 \). Each entry \((i, j)\) of the matrix \( D^{-1} \) can be interpreted as the expected amount of time an individual who is initially introduced in compartment \( j \) spends in compartment \( i \). Thus, we now know how to calculate the expected duration of an infectious period.

The second component of the basic reproductive number is given by \( M \) since the entries of matrix \( M \) represent the rate infections are produced in \( i \) by an individual in compartment \( j \). Therefore, the basic reproductive number, expressed as the product of the expected duration of an infectious period and the rate at which infections occur, is given by \( M D^{-1} x_0 \), where \( M D^{-1} \) is called the next generation matrix evaluated at the disease-free equilibrium. The basic reproductive number,

\[\text{Integrating the matrix } D \text{ simply means integrating each of its individual components with respect to } t.\]
4. Epidemic Theory: A Mathematical Approach

$R_0$, is therefore defined as the spectral radius\(^7\), denoted by $\rho$, of the next generation matrix. Thus, $R_0 = \rho(MD^{-1})$.

### 4.2.5 Example

In this section we use the epidemiological theory developed in the previous sections to solve an SEIR model [48]. A diagram of this model is presented in Figure 4.2.

![Figure 4.2: SEIR model](image)

As we can see from Figure 4.2, the population in this model is divided into four compartments: the susceptible population ($S$), the population of those who are exposed and latently infected ($E$), the infected population ($I$) and the recovered and immunized population ($R$). This model assumes a constant birth rate, $\Pi$ and a mortality rate $\mu$. Thus, individuals enter the system at the rate $\Pi$ and exit the system at the rates $\mu S$, $\mu E$, $\mu I$ and $\mu R$. New exposed individuals arise due to contact between individuals in the $S$ and $I$ compartment at a rate, $\beta SI$. Exposed individuals enter the infected compartment ($I$) at a rate $\kappa E$ and develop immunity to the disease.

---

\(^7\)The spectral radius, $\rho(A)$ of a matrix $A$, is the maximum of the moduli of the eigenvalues of $A$. [48]
and enter the recovered compartment \( R \) at a rate \( \alpha I \). The system of equations for the model is given as follows:

\[
\begin{align*}
S' &= \Pi - \mu S - \beta SI, \\
E' &= \beta SI - (\mu + \kappa)E, \\
I' &= \kappa E - (\mu + \alpha)I, \\
R' &= \alpha I - \mu R
\end{align*}
\]

Now let \( \vec{x} \) represent the subpopulations of the disease compartments, \( E \) and \( I \) and let \( \vec{y} \) represent the subpopulations of the non-disease compartments, \( S \) and \( R \). Thus, we have:

\[
\begin{align*}
\vec{x} &= < x_1, x_2 > \\
\vec{y} &= < y_1, y_2 >
\end{align*}
\]

where, \( x_1 \) represents the population in the exposed compartment \( E \), \( x_2 \) represents the population in the infected compartment \( I \), \( y_1 \) represents the population in the susceptible compartment \( S \) and \( y_2 \) represents the population in the recovered compartment \( R \). Recall that \( F_i \) represents the rate secondary infections increase the population of the \( i \)th disease compartment and \( V_i \) represents the rate disease progression, death and recovery decrease the population of the \( i \)th disease compartment. Thus, we have:

\[
F = \begin{bmatrix}
\beta SI \\
0
\end{bmatrix}
\quad \text{and} \quad
V = \begin{bmatrix}
(\mu + \kappa)E \\
-\kappa E + (\mu + \alpha)I
\end{bmatrix}
\]

Further recall that \( M \) and \( D \) are \( n \times n \) matrices with the \((i, k)\) entry as follows:

\[
M = \frac{\partial F_i}{\partial x_k}(\vec{0}, \vec{y}_0) \quad \text{and} \quad
D = \frac{\partial V_i}{\partial x_k}(\vec{0}, \vec{y}_0)
\]

(4.7)

where \( i, k = 1, 2, ..., n \). Thus, we now get:
4. Epidemic Theory: A Mathematical Approach

\[ M = \begin{bmatrix} 0 & \beta S \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} (\mu + \kappa) & 0 \\ -\kappa & (\mu + \alpha) \end{bmatrix} \]

To compute the next generation matrix, \( MD^{-1} \), we first find \( D^{-1} \) to be:

\[
D^{-1} = \begin{bmatrix}
1 & 0 \\
\frac{1}{(\mu + \kappa)} & \frac{1}{(\mu + \alpha)} \\
0 & \frac{1}{(\mu + \alpha)}
\end{bmatrix}
\]

Thus, the next generation matrix, \( MD^{-1} \) is given as follows:

\[
MD^{-1} = \begin{bmatrix} 0 & \beta S \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix}
1 & 0 \\
\frac{1}{(\mu + \kappa)} & \frac{1}{(\mu + \alpha)} \\
0 & \frac{1}{(\mu + \alpha)}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\frac{\kappa \beta S}{(\mu + \kappa)(\mu + \alpha)} & \frac{\beta S}{(\mu + \alpha)} \\
0 & 0
\end{bmatrix}
\]

Thus, \( R_0 = \rho(MD^{-1}) = \frac{\kappa \beta S}{(\mu + \kappa)(\mu + \alpha)}. \) Therefore, an epidemic will arise if \( \frac{\kappa \beta S}{(\mu + \kappa)(\mu + \alpha)} > 1 \) or the disease will die out if \( \frac{\kappa \beta S}{(\mu + \kappa)(\mu + \alpha)} < 1. \)

In this chapter we have introduced epidemic theory, developed its concepts and discussed the stability of an epidemiological system. In Chapter 5, we will apply these concepts to further discuss the epidemiological model presented by Davidoff et al (2006) [21] and analyze its results by performing numerical simulations of the model.
CHAPTER 5

ANALYSIS OF BASE MODEL

In this section we perform numerical simulations and critically analyze the compartmental demand and supply prostitution model presented by Davidoﬀ et al [21], discussed brieﬂy in Chapter 3, and use it as a base to build our traﬃcking model presented later in Chapter 6. We begin by providing a brief summary of the model, followed by discussing the results of the model simulations. Lastly, we analyze the results and present our modiﬁcations to the current model.

5.1 MODEL SUMMARY

Davidoﬀ et al (2006) [21] uses a compartmental epidemiological system to model the workings of the commercial sex industry, creating two models to study separately the market forces of demand and supply. The ‘female’ model representing the supply of commercial sex workers or prostitutes is divided into four compartments: the susceptible class (S), the commercial sex worker class (P), the class consisting of commercial sex workers who are detained in jail (Jf) and the rehabilitated class (R). The diﬀerential equations representing the female, supply-side model are given as follows: ¹

¹ For a detailed description of this model, please refer to chapter 3.
5. Analysis of Base Model

\[ \dot{S} = \mu_f N - \beta S \frac{P}{N} - \mu_f S \]
\[ \dot{P} = \beta S \frac{P}{N} - (\gamma_f + \mu_f)P + \delta_f \kappa_f J_f + \rho R \frac{P}{N} \]
\[ \dot{J}_f = \gamma_f P - (\kappa_f + \mu_f)J_f \]
\[ \dot{R} = (1 - \delta_f)\kappa_f J_f - \rho R \frac{P}{N} - \mu_f R \]

The ‘male’ model representing the demand for commercial sex workers also consists of four distinct compartments: the population of potential customers, \(C_p\), the population of abstainers, \(G\), the population of active customers, \(C_a\) and the population of the customers detained in jail, \(J_m\). The differential equations representing the male, demand-side model are given as follows:\(^2\)

\[ \dot{C}_p = \Lambda \mu_m M - \eta_1 C_p \frac{C_a}{M} - \eta_2 C_p \frac{G}{M} + \omega C_a + (1 - \delta_m) \kappa_m J_m - \mu_m C_p \]
\[ \dot{G} = (1 - \Lambda) \mu_m M + \eta_2 C_p \frac{G}{M} - \mu_m G \]
\[ \dot{C}_a = \eta_1 C_p \frac{C_a}{M} + \delta_m \kappa_m J_m - (\gamma_m + \omega + \mu_m) C_a \]
\[ \dot{J}_m = \gamma_m C_a - (\kappa_m + \mu_m) J_m \]

In both these models, the ‘disease’ transmitted through the compartments is prostitution, while the ‘infected’ compartments are assumed to be the population of prostitutes and the population of the active customers for the female supply and male demand models respectively. Note that since we assume proportions, we have that \(N = M = 1\).

\(^2\) For a detailed description of this model, please refer to chapter 3.
5.2 Individual Simulation Results

We now run individual simulations using the parameter values used by the authors for each of the models presented in the previous section in order to replicate their results. To simulate these models, we converted the model equations into a set of difference equations which were run on a daily time period, \((t)\). These equations for the female-supply model are given as follows:

\[
S(1,t) = S(1,t-1) + \mu N - \beta S(1,t-1) \frac{P(1,t-1)}{N} - \mu S(1,t-1)
\]

\[
P(1,t) = P(1,t-1) + \beta S(1,t-1) \frac{P(1,t-1)}{N} - (\gamma_f + \mu)P(1,t-1) + \delta_f \kappa_f J_f(1,t-1) + \rho R(1,t-1) \frac{P(1,t-1)}{N} - \mu S(1,t-1)
\]

\[
J_f(1,t) = J_f(1,t-1) + \gamma_f P(1,t-1) - (\kappa_f + \mu)J_f(1,t-1)
\]

\[
R(1,t) = R(1,t-1) + (1 - \delta_f)\kappa_f J_f(1,t-1) - \rho R(1,t-1) \frac{P(1,t-1)}{N} - \mu R(1,t-1)
\]

The difference equations for the male-demand model are given as follows:

\[
C_p(1,t) = C_p(1,t-1) + \lambda \mu M - \eta_1 C_p(1,t-1) \frac{C_a(1,t-1)}{M} - \eta_2 C_p(1,t-1) \frac{G(1,t-1)}{M}
\]

\[
+ \omega C_a(1,t-1) + (1 - \delta_m) \kappa_m J_m(1,t-1) - \mu C_p(1,t-1)
\]

\[
G(1,t) = G(1,t-1) + (1 - \lambda) \mu M + \eta_2 C_p(1,t-1) \frac{G(1,t-1)}{M} - \mu G(1,t-1)
\]

\[
C_a(1,t) = C_a(1,t-1) + \eta_1 C_p(1,t-1) \frac{C_a(1,t-1)}{M} + \delta_m \kappa_m J_m(1,t-1) - (\gamma_m + \omega + \mu) C_a(1,t-1)
\]

\[
J_m(1,t) = J_m(1,t-1) + \gamma_m C_a(1,t-1) - (\kappa_m + \mu) J_m(1,t-1)
\]

The values used for the simulations are presented in Tables 5.1 and 5.2 [21]. The values given in these tables are taken directly from Davidoff et all (2006) [21] and represent annual rates. They have been divided by 365 in our simulations (with the

\[\text{Please refer to the appendix for the complete MATLAB code used.}\]
5. Analysis of Base Model

exception of $\delta_f$ and $\delta_m$ since they represent proportions) in order to be able to look at the changes in the various populations on a daily basis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_f$</td>
<td>Birth/death rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Recruitment rate of commercial sex workers</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma_f$</td>
<td>Arrest rate of commercial sex workers</td>
<td>1.37</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Relapse rate of rehabilitated commercial sex workers</td>
<td>3</td>
</tr>
<tr>
<td>$\kappa_f$</td>
<td>Rate of leaving jail</td>
<td>15</td>
</tr>
<tr>
<td>$\delta_f$</td>
<td>Proportion of jailed workers who immediately resume working</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 5.1: Parameter values for the female supply model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_m$</td>
<td>Birth/death rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>Proportion of the male population that represents potential customers</td>
<td>0.9</td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>Recruitment rate of potential to active customers</td>
<td>30</td>
</tr>
<tr>
<td>$\eta_2$</td>
<td>Recruitment rate of potential customers to abstainers</td>
<td>0.25</td>
</tr>
<tr>
<td>$\gamma_m$</td>
<td>Arrest rate of active customers</td>
<td>0.25</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Rate of ceasing activity due to an increase in arrests</td>
<td>0.5</td>
</tr>
<tr>
<td>$\kappa_m$</td>
<td>Rate of leaving jail</td>
<td>15</td>
</tr>
<tr>
<td>$\delta_m$</td>
<td>Proportion of jailed customers that resume activity</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Table 5.2: Parameter values for the male demand model

Using these values, we found that the majority of the movement in the male-demand model, specifically in the populations of the potential and active customers,
5. Analysis of Base Model

happens within approximately 150 days, i.e., less than a year. Starting at 5%, the active customer population steeply rises to about 85% of the entire population in approximately 3 months or 90 days. On the other hand, the population of potential customers drops from 80% of the population to about 1% of the population in the same amount of time. However, there is not much of a change seen in the populations of both the jailed as well as the abstainers. The movement in the populations of the male model are shown in Figure 5.1.

![Figure 5.1: Male-demand model](image)

Looking now at the female-supply model, we observe that the populations move much slower than the populations in the male model. Given the same time frame of 150 days, it is evident from Figure 5.2 that even though there was a surge in
demand for prostitution, (with almost the entire male population becoming active customers), the population of prostitutes did not encounter significant change, with an increase from an initial population of 10% to a population of 13%. Similarly, the other populations saw very slight movement, with a change of less than 1 percentage point in each compartment.

![Female-supply model](image)

**Figure 5.2:** Female-supply model

The steady state\(^4\) conditions for both models are given in Figure 5.3 and Figure 5.4. The male model reaches approximately 5% of accuracy of the steady state population value in less than a year, while the female model reaches an accuracy of

\(^4\)Steady state is a stable state that populations reach where they do not change with time, but remain a constant.
5% of its steady state population value after about 4500 iterations or approximately 12 years.

As we can see from Figure 5.3, there is an extremely insignificant amount of change between the populations after the first 3 months and the steady state populations of the male model with just a slight increase in the population of the abstainers and a slight decrease in the population of the active customers. Thus, according to this model, about 86% of the entire male population will become active customers during their lifetime and only about 14% of the population does not take part in prostitution at all. The results also imply that not only does 86% of the entire
population become active customers, the change from potential to active happens within the first 3 months. This does not concur with current literature and statistics regarding the male demand for prostitution and will further be addressed in sections 5.4 and 5.5 [3]. Even if we take a high estimate of the worst countries in the world in terms of the percentage of its population that buy prostitution services, 86% is too high a percentage to be an accurate representation of the workings of the real world. The steady state values for each population are given in Table 5.3.

In the female model we see a lot of movement happening within the first 8 years. Thus, the female model has a much slower and consistent pace of change. The model reaches a steady state after approximately 12 years.

![Female model at steady state](image-url)
From Figure 5.4, we can see that about 67% of the women are active, working prostitutes, 6% are jailed, 22% in rehabilitation centers and only about 3% of the entire population of females in the susceptible compartment. This implies that about 96% of the entire population takes part in prostitution at least once during their lifetime. Another implication of this result is that about 67% of the female population is required in order to meet the demand of about 86% of the male population. Thus, each prostitute serves an average of 1.5 customers. All of these results contradict current literature and are an inconsistent representation of the workings of world of prostitution. The exact values for the steady populations are given below in Table 5.3 are at accurate at 5%.

<table>
<thead>
<tr>
<th>Population</th>
<th>Initial value</th>
<th>Value at 150 days</th>
<th>Steady state value</th>
<th>Days taken to reach steady state</th>
</tr>
</thead>
<tbody>
<tr>
<td>Susceptible females</td>
<td>0.8</td>
<td>0.7629</td>
<td>0.03</td>
<td>4,500</td>
</tr>
<tr>
<td>Prostitutes</td>
<td>0.1</td>
<td>0.1342</td>
<td>0.67</td>
<td>4,500</td>
</tr>
<tr>
<td>Jailed prostitutes</td>
<td>0.05</td>
<td>0.0121</td>
<td>0.06</td>
<td>4,500</td>
</tr>
<tr>
<td>Rehabilitated prostitutes</td>
<td>0.05</td>
<td>0.908</td>
<td>0.22</td>
<td>4,500</td>
</tr>
<tr>
<td>Potential customers</td>
<td>0.8</td>
<td>0.0202</td>
<td>0.02</td>
<td>150</td>
</tr>
<tr>
<td>Abstainers</td>
<td>0.1</td>
<td>0.1024</td>
<td>0.10</td>
<td>150</td>
</tr>
<tr>
<td>Active customers</td>
<td>0.05</td>
<td>0.8632</td>
<td>0.86</td>
<td>150</td>
</tr>
<tr>
<td>Jailed customers</td>
<td>0.05</td>
<td>0.0142</td>
<td>0.01</td>
<td>150</td>
</tr>
</tbody>
</table>

Table 5.3: Different values of each population at two time periods.

Although these results are an accurate replication of the results presented in Davidoff et al (2006), they are unreliable as they are too high an estimate. The reason we find the population of active customers growing at such a fast rate and recruiting
almost the entire population as well as the population of prostitutes recruiting the vast majority of the population is due to the choice of parameters used in the model by the authors. For example, assuming that each active customer recruits about 30 active customers per year, will eventually lead to the entire population being recruited to active customers, which is what is seen in these results. A similar argument can be made for the recruitment of the population of prostitutes. These factors are taken into account and further discussed and modified in sections 5.4 and 5.5.

5.3 Coupled Simulation Results

In this section, we simulate and analyze the author’s coupled model, which changes parameters to make them interactive between both the demand and supply models. Since some of the parameters now depend on both models, these are not constant but rather dependent on other variables in one or both of the models. These parameters must change with each iteration of the coupled model. The changes in the parameters are discussed in the proceeding section.

5.3.1 Changes in Parameters

The first change made by the author is in the recruitment rate of susceptible women to prostitutes in the female model. In the author’s previous model, the recruitment rate was merely a constant, $\beta$, which had a fixed value. In the coupled model, the recruitment rate of prostitutes reflects the demand for prostitutes from male customers. In addition, the recruitment rate also depends on the profit made by each prostitute as a high number of prostitutes would imply a lower level of profit per prostitute. Thus, the new recruitment rate is given as follows:
5. Analysis of Base Model

\[ \beta = \beta_0 k_m \frac{C_a}{P} \]

where,
\[ \beta_0 = \text{scaling factor}\left(\frac{\text{Prostitutes}}{\text{transactions}}\right) \]
\[ k_m = \text{Maximum sexual activity desired by an average male customer}\left(\frac{\text{transactions}}{\text{time}}\right) \]
\[ C_a = \text{proportion of active customers.} \]
\[ P = \text{proportion of prostitutes.} \]

This formula now ensures that the recruitment moves in accordance with the movement of the different populations involved. For example, if the population of active customers, \( C_a \) increases, this will lead to an increase in the recruitment rate, \( \beta \).

An analogous argument is given for \( \rho \), the recruitment rate of prostitutes from the rehabilitated class, or the relapse rate. Thus, the relapse rate is now given as follows:

\[ \rho = \rho_0 k_m \frac{C_a}{P} \]

where,
\[ \rho_0 = \text{scaling factor}\left(\frac{\text{Prostitutes}}{\text{transactions}}\right) \]
\[ k_m = \text{Maximum sexual activity desired by an average male customer}\left(\frac{\text{transactions}}{\text{time}}\right) \]
\[ C_a = \text{population of active customers.} \]
\[ P = \text{population of prostitutes.} \]

The change in relapse rate is the same as the change in the recruitment rate of new prostitutes. Thus, this rate also moves in accordance with the populations involved. For example, if the population of prostitutes, \( P \), were to increase, this would lead to a decrease in the relapse rate, \( \rho \).

The next change is seen in the recruitment rate, \( \eta_1 \), of potential to active customers in
the male model. Earlier a constant, the recruitment rate now depends on the number of prostitutes available to customers and also takes into account that small decreases in supply does not affect the demand for prostitution. This new recruitment rate is now given as follows:

\[ \eta_1 = \frac{\eta_m P}{P + \epsilon N} \]

where,

- \( \eta_m \) = Maximum recruitment rate
- \( \epsilon \) = scaling factor for the saturation of supply, where \( \epsilon \ll 1 \)
- \( P \) = proportion of prostitutes.
- \( N \) = proportion of females in the system.

This rate is mostly dependent on \( \eta_m \), the maximum recruitment rate of active customers. Since \( \epsilon \) is much smaller than 1 and therefore, close to 0, we have that \( P \) is just slightly smaller than the denominator, \( P + \epsilon N \). Thus, \( \frac{P}{P + \epsilon N} \) would always be close to 1. Therefore, the population of prostitutes has a smaller impact on \( \eta_1 \) than \( \eta_m \).

The next change in parameters is seen in the arrest rate for active customers, \( \gamma_m \). The new arrest rate now reflects the increase in legal enforcement as a result of an increase in the activity by customers. The rates are now given as follows:

\[ \gamma_m = \frac{\sigma_m C_a}{M} \]

where,

- \( \sigma_m \) = scaling factor for the prevalence of active customers on the arrest rate
- \( C_a \) = proportion of active customers.
- \( M \) = proportion of males in the system.

An analogous argument is given for \( \gamma_f \), the the arrest rate of prostitutes which now
reflects the increase in legal enforcement as a result of an increase in activity by prostitutes. Thus, the new arrest rate is now given as follows:

\[ \gamma_f = \frac{\sigma_f P}{N} \]

where,
\( \sigma_f \) = scaling factor for the prevalence of prostitutes on the arrest rate
\( P \) = proportion of prostitutes.
\( N \) = proportion of females in the system.

Both the arrest rates, \( \gamma_m \) and \( \gamma_f \), depend significantly on the population of active customers and population of prostitutes respectively. For example, an increase in these populations would cause a significant increase the respective arrest rates.

The last change is seen in \( \omega \), which is the rate of ceasing activity due to the knowledge of an increase in the arrests of active customers. The new rate reflects the fact that the knowledge of arrests usually travels through the word of mouth, thus implying a slower movement. It is given as follows:

\[ \omega = \frac{\omega_0 J_m}{C_a + C_p + J_m} \]

where,
\( \omega_0 \) = scaling factor
\( C_a \) = proportion of active customers.
\( C_p \) = proportion of potential customers.
\( J_m \) = proportion of jailed customers.

The rate of temporarily ceasing activity, \( \omega \), depends on almost all the compartments in the male demand model. Thus, a change in any of these compartments would create a change in the \( \omega \). For example, if the populations of active potential customers increase, it would cause a decrease in the rate of temporarily ceasing activity.
5. Analysis of Base Model

The author does not provide any values for the new constants used in the changed formulas for the parameters. Thus, we have assigned values to these constants which keep the parameters close to their original values as given in Table 5.1 and Table 5.2. The values assigned to these constants are given in Table 5.4.

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>$\frac{1}{365}$</td>
</tr>
<tr>
<td>$k_m$</td>
<td>$\frac{50}{365}$</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>$\frac{3}{365}$</td>
</tr>
<tr>
<td>$\eta_m$</td>
<td>$\frac{50}{365}$</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.001</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>$\frac{0.3}{365}$</td>
</tr>
<tr>
<td>$\sigma_f$</td>
<td>$\frac{1.5}{365}$</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>$\frac{0.5}{365}$</td>
</tr>
</tbody>
</table>

**Table 5.4:** Values for the constants in the new parameter equations

5.3.2 Results of the Coupled Model

After running simulations for the coupled model, we found that the results of this model were very similar to the results of the previous model. Once again, the majority of the movement in the male model takes place at an extremely rapid rate, occurring within the first 3 months or 90 days. There is very little change in the populations of abstainers and jailed customers, but the populations of active potential customers see a large change. This can be seen in Figure 5.5.
As seen from Figure 5.5, there is a steep rise in the demand for prostitution which leads the population of active customers to rise from an initial population of 5% to a population of about 88%. On the other hand, to accommodate this rise in demand, the population of potential customers falls sharply from an initial population of 80% to a population of about 0.2%. Jailed customers see a slight drop to about 0.1% whereas there is almost no change in the population of abstainers which stays steady at 10%. Once again, although these results accurately replicate the results presented by the author, an active customer population of 88% among other things is extremely unrealistic and does not concur with current literature [3].

The results of the coupled female model in the first 90 days were found to be very
similar to the results of the female populations for the first 150 days in the single model. As can be seen from Figure 5.6, once again, for the same time frame, we did not find any significant change in the different population sizes as compared to their initial values. Even though, there was a steep rise in the demand for prostitutes, the population of prostitutes only increased by around 4 percentage points. The population of susceptible women decreased by 2 percentage points, the population of jailed prostitutes decreased by approximately 0.3 percentage points and finally, the population of rehabilitated workers saw an increase by about 2 percentage points.

Figure 5.6: Female populations for the coupled model at 90 days
In the coupled model, we found that both the male and female populations reach a steady state at approximately the same time as seen in the previous model. The steady state populations for the male model (accurate to 2 decimal places) are given below in Figure 5.7. As can be seen from the graph, there is almost no change in any of the four populations after the first 90 days. The results of this model are very similar to the results of the previous model in that they both show an extremely rapid rate of increase initially and both models have about the same sizes for all four populations. In the coupled model as well, a very high number of males are assumed to be actively involved in prostitution during their life time, with the population of active customers settling in at the extremely high proportion of 88% of the entire population.

![Figure 5.7: Male populations at steady state](image-url)
5. Analysis of Base Model

On the other hand, the results of the female model are similar to the previous model in that they exhibit more stable behaviour. But there is a stark difference between the population sizes of the previous model and the population sizes of the coupled model, especially for the populations of the susceptible women and prostitutes. Another different between the models is that they both take a vastly different time to reach a steady state, with the previous model taking approximately 12 years and coupled model taking approximately 30 years or 11,000 iterations (accurate at 2 decimal points). The results of this model can be seen in Figure 5.8.

![Figure 5.8: Female populations at steady state](image)

As we can see, to meet the the rise in demand for prostitution services, the population of prostitutes increases from an initial population of 10% to a population of approximately 44%. This is much lower than the size of the population of
prostitutes in the previous model which was stable at approximately 67%. Although, the estimate of the coupled model adheres more to current literature, it is still too high to be assumed to be a real estimate of the workings of the prostitution world. The population of susceptible women, on the other hand, is significantly larger in the coupled model than in the single model. We found a steady fall in the population of susceptibles to accommodate the population of prostitutes, which ultimately settled at approximately 17%, much higher than the population in the previous model, which stayed steady at about 3.5%. We also found a large increase in the population of rehabilitated workers, which rises from an initial 5% to a population size of about 37%. Finally, the jailed workers initially see a slight drop in numbers, but increase and stay steady at a final population of approximately 2%. The values for the steady state population sizes of the female model are given in Table 5.5.

<table>
<thead>
<tr>
<th>Population</th>
<th>Initial value</th>
<th>Value at 90 days</th>
<th>Steady state value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Susceptible females</td>
<td>0.8</td>
<td>0.7842</td>
<td>0.1796</td>
</tr>
<tr>
<td>Prostitutes</td>
<td>0.1</td>
<td>0.1413</td>
<td>0.4359</td>
</tr>
<tr>
<td>Jailed prostitutes</td>
<td>0.05</td>
<td>0.0029</td>
<td>0.0190</td>
</tr>
<tr>
<td>Rehabilitated prostitutes</td>
<td>0.05</td>
<td>0.0716</td>
<td>0.3656</td>
</tr>
<tr>
<td>Potential customers</td>
<td>0.8</td>
<td>0.0024</td>
<td>0.0023</td>
</tr>
<tr>
<td>Abstainers</td>
<td>0.1</td>
<td>0.1014</td>
<td>0.1022</td>
</tr>
<tr>
<td>Active customers</td>
<td>0.05</td>
<td>0.8811</td>
<td>0.8801</td>
</tr>
<tr>
<td>Jailed customers</td>
<td>0.05</td>
<td>0.0151</td>
<td>0.0155</td>
</tr>
</tbody>
</table>

Table 5.5: Different values of each population at two time periods.
5.4 **Remarks**

Although Davido et al. (2006) [21] presents a good theoretical base to model the dynamics of the prostitution world, the results of both the single and coupled model lead to the conclusion that both these models are inherently flawed in their process of assigning values to their parameters. This leads to results that are unreliable. The fundamental mistake made by the author is choosing parameter values which are inconsistent with one another, specifically, values which represent both proportions of a certain population as well as numbers of individuals. For example, in the female model, the value of $\delta_f$ is 0.5. This represents the proportion of jailed prostitutes that immediately resume working. Thus, the authors assume that 50% of jailed workers resume being a prostitute once their term has ended. On the other hand, the value of $\beta$, the recruitment rate of susceptible women to prostitutes, is 1. Thus, according to the author, this implies that one new prostitute is recruited by each current prostitute each year. In this example, the author is confusing proportions with actual numbers of individuals, and also choosing unrealistic values. Assuming that each prostitute recruits one additional prostitute per year is reasonable if the population of current prostitutes is small. But if the size of the population is large, even a 5% population of current prostitutes would recruit the majority of the population into prostitution. Similarly, in the male model, the value of $\eta_1$, the recruitment rate of potential to active customers, is 30, which implies that each active customer influences 30 potential customers to take part in prostitution services and therefore become an active customer. Whereas the value of $\delta_m$ is 0.7, which implies that 70% of detained customers immediately resume activity. Combining both proportions and individual estimates is problematic for two reasons. First of all, since the model assumes a constant population, the value of each population represents a proportion. For example, if $C_p$ is 0.8, this means that 80% of the entire population represents potential customers. Thus, when you combine proportions with actual numbers of
individuals, the results produced by the model are dubious. For example, when we let $\eta_1$ be 30, the number of individuals recruited from potential to active customers would be extremely different depending on the size of the population that this number is drawn from. If the population was 100 individuals, this would mean that each active customer recruits 30% of this population. If the population was 10,000 individuals, it would imply that each active customer recruits 0.3% of the population to become an active customer. Thus, the implications of this rate change drastically when we combine proportions and individual numbers to calculate the transfer of flow from one compartment to another. Second of all, some of the parameter values chosen are extremely high and unrealistic. For example, assuming that each active customer recruits 30 individuals from the population of potential customers is not a practical assumption. Such assumptions lead to unreliable behaviour in the model such as the extremely rapid, high rate of growth seen in the active customer population in the male model which lead to almost 90% of the population becoming active customers.

Thus, to make the model results produce realistic estimates of the real world population sizes of these groups, we have implemented some changes that cause the model to be more feasible and produce results more in line with information found in the literature. These changes as well as their results are discussed in the proceeding sections.
5. Analysis of Base Model

5.5 Modified Model

The focal point of this section is to build a supply and demand model for prostitution, with parameter values that produce reasonable results. Thus, we are not overly concerned with results and population sizes that very precisely replicate the real population sizes in the prostitution market. We will then analyze the effect of introducing a human trafficking component to see how trafficking alters our results. This will be done in Chapter 6. In this section we discuss the modifications made in the demand-supply model presented by Davidoff et. al (2006) [21].

For both the female and male parameters, all values have been converted to proportions and have been divided by a factor of 10 or 100 depending on its original value. This removes the problem of having inconsistent units for each parameter and also removes the problem of having values that are unrealistically high. The final change has been made in the arrest rates of both prostitutes ($\gamma_f$) and active customers ($\gamma_f$). These are rates were originally given as the following:

$$
\gamma_m = \frac{\sigma_m C_a}{M} \quad \text{and} \quad \gamma_m = \frac{\sigma_f P}{N}
$$

where,

$\sigma_m = \text{scaling factor for the prevalence of active customers on the arrest rate}$

$\sigma_f = \text{scaling factor for the prevalence of prostitutes on the arrest rate}$

$C_a = \text{proportion of active customers.}$

$P = \text{proportion of prostitutes.}$

$M= \text{proportion of males in the system.}$

$N= \text{proportion of females in the system.}$

However, since these rates do not depend on and interact with variables from both the demand and supply models, we have converted these rates back to constants.
The new values assigned to the parameters (which represent daily rates) are given in Tables 5.6 and Table 5.7.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_f$</td>
<td>Birth/death rate</td>
<td>$0.025 \frac{1}{365}$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Recruitment rate of commercial sex workers (initial value)</td>
<td>$1 \frac{1}{365}$</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>scaling factor $\left(\frac{\text{Prostitutes}}{\text{transactions}}\right)$</td>
<td>$0.1 \frac{1}{365}$</td>
</tr>
<tr>
<td>$k_m$</td>
<td>Maximum sexual activity desired by an average customer</td>
<td>$50 \frac{1}{365}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Relapse rate of rehabilitated commercial sex workers (initial value)</td>
<td>$3 \frac{1}{365}$</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>scaling factor $\left(\frac{\text{Prostitutes}}{\text{transactions}}\right)$</td>
<td>$0.3 \frac{1}{365}$</td>
</tr>
<tr>
<td>$\gamma_f$</td>
<td>Arrest rate of commercial sex workers</td>
<td>$0.0137 \frac{1}{365}$</td>
</tr>
<tr>
<td>$\kappa_f$</td>
<td>Rate of leaving jail</td>
<td>$0.15 \frac{1}{365}$</td>
</tr>
<tr>
<td>$\delta_f$</td>
<td>Proportion of jailed workers who immediately resume working</td>
<td>$0.5$</td>
</tr>
</tbody>
</table>

*Table 5.6: Parameter values for the female supply model*
Using these new values, we now perform simulations on the model and analyze and compare the results.
5.5.1 Results of the Modified Model

After making the necessary changes, we found that the model produced vastly different results than those produced in the Davidoff model [21]. Figure 5.9 shows the behaviour of the modified male model over the first 5000 days. As we can see from the graph, the population of potential and active customers change at a much slower rate in this model.

![Figure 5.9: Male populations over 5000 days](image)

The populations begin to approach a steady state after about 15,000 iterations or approximately 40 years. The population of active customers initially increases and reaches a maximum value of 59.4% after which it has a slight decrease and settles in at approximately 51% of the population. This estimate seems much more reliable
than the ones from the previous model where almost 90% of the population was seen to be recruited into being an active customer. The population of potential customers is seen to decrease and settle at approximately 26.5% of the population. Lastly, the population of abstainers and jailed customers also see a slight increase, with a steady state value of about 21% and 0.85% respectively. These results are graphically shown in Figure 5.10.

![Figure 5.10: Male populations at steady state](image)

Looking at the results of the female model, we only found one major difference between the results of this model and the results of the previous models. Once again, the female populations are seen to grow at an extremely steady pace, and start to approach a steady state after 11,000 iterations or approximately 30 years.
5. Analysis of Base Model

The only difference in this model is that the population of prostitutes increases just a little from its initial value of 10% to a steady state value of approximately 18%. The population of susceptible decreases from a size of 85% to about 78%. Finally, the populations of the jailed and rehabilitated workers see very little change with steady state values of 1% and 2% respectively. These estimates seems to be more realistic than the ones from the previous models which saw almost 50% of the entire population becoming prostitutes. The results of the female model can be graphically seen in Figure 5.11 below.

![Figure 5.11: Female populations at steady state](image)
The exact steady state values for both the male and female models are given in Table 5.8.

<table>
<thead>
<tr>
<th>Population</th>
<th>Initial value</th>
<th>Steady state value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Susceptible females</td>
<td>0.85</td>
<td>0.7808</td>
</tr>
<tr>
<td>Prostitutes</td>
<td>0.1</td>
<td>0.1818</td>
</tr>
<tr>
<td>Jailed prostitutes</td>
<td>0.02</td>
<td>0.0142</td>
</tr>
<tr>
<td>Rehabilitated prostitutes</td>
<td>0.03</td>
<td>0.0232</td>
</tr>
<tr>
<td>Potential customers</td>
<td>0.55</td>
<td>0.2657</td>
</tr>
<tr>
<td>Abstainers</td>
<td>0.1</td>
<td>0.2134</td>
</tr>
<tr>
<td>Active customers</td>
<td>0.349</td>
<td>0.5124</td>
</tr>
<tr>
<td>Jailed customers</td>
<td>0.001</td>
<td>0.0085</td>
</tr>
</tbody>
</table>

Table 5.8: Different values of each population at steady state.

The results of the modified model are deemed to be much more realistic and reliable than the results of the model presented by Davidoff et al(2006) [21]. The rapid growth that made the male populations reach a steady state within one year, as seen in the previous models, is now replaced with a much slower and steady pace of growth. In addition, we do not have an unrealistically high estimate of the number of men who purchase prostitution services. The estimates of the modified model, are much closer to the estimates of average male customer population found across the world, mostly due to the choice of the parameter values. For example, according to an estimate, the percentage of men who indulge in prostitution services in Italy ranges from 17 - 45%, while in countries like Australia and Cambodia, estimates may fall around 20% and as high as 70% respectively [3] [14] [32]. Thus, the estimate of the modified model of about 51% is a good average estimate of the current state of affairs.
Similarly, the female models see a much better estimate of the various populations. Data on the percentage of prostitutes across countries is often contradictory and thus, often unreliable. For example, according to some estimates, there are about 76,000 prostitutes in Thailand, whereas according to other estimates, the number of prostitutes found in Thailand reach up to 200,000. [16] Data is often difficult to collect on the proportion of the population involved in prostitution as a large majority of the industry is underground. However, an 18% population of prostitutes (as estimated by the modified model) can be considered to be more realistic than an estimate of about 67% (estimated by the previous models). In addition, the ratio of the number of proportion of active customers to the proportion of prostitutes has seen an increase from about 1 in the original models, to about 3 in the modified models. The latter ratio is more reliable as it adheres more with the current literature. The number of customers per prostitute is hard to estimate, but data on the average number of clients per day vary from about 5 to as high as 15, depending on regulations involving the industry [46] [7] [33]. Since a portion of these customers may be repeat customers, an estimate of about a ratio of 3, as produced by the modified model, although on the lower side, seems reasonable.

We now have a complete model which can be used as a base for comparing and analyzing results. The estimates made by this model are far better representations of the workings of the real world. In the next chapter, we will introduce human trafficking into our base model and use these parameters to analyze the changes caused by the introduction of a trafficking element.
CHAPTER 6

HUMAN TRAFFICKING MODEL

In this chapter, we introduce human trafficking into the modified model discussed at the end of Chapter 5, perform simulations\(^1\) and finally analyze the results. We begin by analyzing a scenario in which prostitution is illegal and then move onto situations involving different levels of the legalization of prostitution and analyze its effects on the population of trafficked individuals. We then draw conclusions from our results to determine the effect of the legalization of prostitution on the market for human trafficking.

6.1 INTRODUCING HUMAN TRAFFICKING IN THE MODEL

We continue to use the modified version of the model discussed in the previous chapter, but adjust it to include the element of human trafficking. Since we are looking into sex trafficking which only affects the supply side of the model, trafficking is introduced in our model by stratifying the population of prostitutes into two separate groups: voluntary prostitutes and trafficked prostitutes. Our female model now has five compartments. This includes susceptible women (S), voluntary prostitutes (VP), trafficked prostitutes (TP), jailed workers (J\(_f\)) and rehabilitated

\(^1\)Please refer to the appendix for the MATLAB code used.
workers (R). The flow of individuals from one component to another is graphically shown in Figure 6.1.

We begin by looking at the case in which prostitution is illegal. Individuals enter the system at the rate of $\mu_f N$, where $\mu_f$ is the birth rate and $N$ represents the proportion of individuals in the system. We assume a constant population, thus the per capita birth rate is equal to the per capita exit rate. Individuals exit each compartment at the rate of $\mu_f S, \mu_f VP, \mu_f TP, \mu_f J_f$ and $\mu_f R$. In addition, we also assume that the individuals involved in prostitution are between the ages of 10 - 50 years, thus we let $\frac{1}{\mu_f}$ to be approximately 40. After entering the system, susceptible females either work as prostitute by choice at the rate of $\beta_{VP} S \frac{P}{N}$, or are trafficked into prostitution at a rate of $\beta_{TP} S \frac{P}{N}$, where $\beta_{VP}$ and $\beta_{TP}$ represent the recruitment rate of voluntary prostitutes and the rate at which susceptible women are trafficked into prostitution.
respectively. The recruitment rate of prostitutes depends on both the demand for prostitute services from male customers, as well as the profit made by each prostitute. Thus, the recruitment rates assume the forms \( \beta_{VP} = \beta_1 k_m \frac{C_a}{VP + TP} \) and \( \beta_{TP} = \beta_2 k_m \frac{C_a}{VP + TP} \). Here, \( \beta_1 \) and \( \beta_2 \) represent the scaling factor for the recruitment of voluntary prostitutes and trafficked prostitutes respectively, and \( k_m \) once again represents the maximum sexual activity desired by an average male customer. Since it is more common to have trafficked prostitutes than voluntary prostitutes, we assume that \( \beta_2 > \beta_1 \). After entering the world of prostitution, both voluntary prostitutes as well as trafficked prostitutes are arrested at the rate of \( \gamma_f VP \) and \( \gamma_f TP \) respectively. Since legal authorities arrest both voluntary and trafficked prostitutes without a distinction, we use the same value for the arrest rate, \( \gamma_f \) for both the groups of prostitutes in our model. Once arrested, individuals either choose to go to rehabilitation centers at a rate of \( (1 - \delta_f) \kappa_f J_f \), or voluntarily choose to go back to working as a prostitute at the rate of \( \delta_f \kappa_f J_f \), where \( \delta_f \) represents the proportion of jailed prostitutes who immediately resume working and \( \kappa_f \) represents the rate of leaving the detention center. Finally, if individuals choose to go to the rehabilitation class, they may relapse and choose to continue working as a prostitute again at a rate of \( \rho R_{VP} \), where \( \rho \) represents the rate of relapse. We do not include any transfer of individuals from the jailed and rehabilitation classes to the trafficked prostitute class, since we assume that if a woman chooses to resume working as a prostitute, it is by her choice. Thus, she now becomes a voluntary prostitute. The equations for our supply-side (female) model are thus, given as follows:
The diagram of the transfer of individuals from one compartment to another in the male model is given in Figure 6.2 [21]. There has not been any significant change made in this model compared to the modified model discussed in section 5.5.

![Diagram of the male model](image)

**Figure 6.2:** Diagram of the male model

As we can see from Figure 6.2, the male demand model still has four compartments. This includes potential customers ($C_p$), abstainers ($G$), active customers ($C_a$) and
jailed customers ($J_m$). Individuals enter the class of potential customers at a rate
of $\Lambda \mu_m M$. Here, $\Lambda$ is the proportion of the male population that is mature enough
to be a potential customer, $\mu_m$ is the birth rate and $M$ represents the proportion of
all individuals in the system. Individuals can also enter the class of permanent
abstainers at a rate of $(1 - \Lambda) \mu_m M$. Once again, we assume a constant population,
thus the per capita birth rate is equal to the per capita exit rate. Individuals exit
each compartment at the rate of $\mu_m C_p, \mu_m G, \mu_m C_a$ and $\mu_m J_m$. In addition, we also
assume that the individuals involved in the purchase of prostitution services are
between the ages of 20 - 60 years, thus we let $\frac{1}{\mu_m}$ to be approximately 40. We
do not allow any transfer of individuals from the abstainer compartment to any
other compartments since we assume that individuals who enter the abstainer class
refrain from indulging in prostitution services permanently. On the other hand,
once individuals enter the class of potential customers, they can either move to the
class of active customers at a rate of $\eta_1 C_p \frac{C_a}{M}$, where $\eta_1$ represents the recruitment
rate of active customers, or move to the class of abstainers at a rate of $\eta_2 C_p \frac{G}{M}$,
where $\eta_2$ represents the recruitment rate of abstainers. Since we have stratified the
population of prostitutes into two separate groups, the recruitment rate of active
customers has now been modified to reflect the inclusion of trafficked prostitutes.
Thus, it assumes the form $\eta_1 = \frac{\eta_m (VP + TP)}{(VP + TP) + \epsilon N}$. Active customers are arrested at
a rate of $\gamma_m C_a$, where $\gamma_m$ represents the arrest rate. Due to an increase in arrests,
some active customers may temporarily cease activity owing to the risk of being
arrested themselves. This will lead to a transfer of individuals from the class of
active customers back to the class of potential customers, at a rate of $\omega C_a$, where $\omega$
represents the rate of ceasing activity due to the knowledge of arrests. Finally, once
customers complete their jail term, they can choose to resume activity at a rate of
$\delta_m \kappa_m J_m$, where $\delta_m$ represents the proportion of customers who resume activity and
$\kappa_m$ represents the rate of leaving jail, or they can choose to temporarily cease activity
at a rate of \((1 - \delta_m)\kappa_m J_m\). The equations for the male demand model are given as follows:

\[
\dot{C}_p = \Lambda \mu_m M - \eta_1 C_p \frac{C_a}{M} - \eta_2 C_p \frac{G}{M} + \omega C_a + (1 - \delta_m)\kappa_m J_m - \mu_m C_p
\]

\[
\dot{G} = (1 - \Lambda)\mu_m M + \eta_2 C_p \frac{G}{M} - \mu_m G
\]

\[
\dot{C}_a = \eta_1 C_p \frac{C_a}{M} + \delta_m \kappa_m J_m - (\gamma_m + \omega + \mu_m) C_a
\]

\[
\dot{J}_m = \gamma_m C_a - (\kappa_m + \mu_m) J_m
\]

### 6.2 Types of Regulations

We now perform simulations on our trafficking model and analyze the results for 3 different types of regulation. We conclude with a comparison of our results, using the first case (illegal prostitution) as our base, and provide recommendations for legal authorities.

#### 6.2.1 Illegal Prostitution: India & Nepal

The first type of regulation we analyze is one in which prostitution is illegal. Such regulation can be found in countries like India and Nepal. [11] [3] [34] In these countries brothel ownership and ‘pimping’ is considered illegal. It is more common to arrest the woman providing the service rather than the man buying the service [34]. This does very little to curb the demand for prostitution despite it being illegal. Thus, regardless of the regulation, prostitution is highly rampant in both India and Nepal. In both these countries, women can be found entering the business voluntarily due to financial pressure, but the majority of women involved in prostitution are trafficked. India is often considered to be a global hub of sex trafficking due to its high demand, especially from major metropolitan cities such
6. Human Trafficking Model

as Mumbai and Kolkata. These cities are seen as the dumping ground for most of the trafficked women coming in from Nepal [29] [11] [35]. Once in the business of prostitution, it is extremely rare to exit, since escape is difficult and has immensely harsh consequences [29] [10]. Brothel owners also usually have arrangements with local authorities, thus the legal enforcement does not provide any efficient assistance. Even if arrests are made, it is usually the prostitute who is criminalized and they are usually sent back to their owners through the payment of bribes. Successful rehabilitation is also a rare phenomenon. Due to the physical and emotional abuse and torture as well as the ostracism from society, women are often found to relapse back into the profession regardless of rehabilitation programs [35] [5].

Under these assumptions, we have used reasonable parameters to perform simulations on our trafficking model. The values used for each parameter for the female model have not been changed from those of the modified model in section 5.5. Two additional values have been added for the recruitment rate of trafficked prostitutes, $\beta_{TP}$ and the scaling factor of trafficked prostitutes, $\beta_2$. Both these values have are close to the values of their counterparts, $\beta_{TP}$ and $\beta_2$. The only difference is that both the values representing trafficking rates are higher than those representing voluntary prostitution, since it is assumed that prostitutes are more often trafficked than voluntary. These values are given below in Table 6.2.1. The values for the parameters of the male model are the same as the ones discussed in section 5.5 for our modified model and can be found in Table 5.7.
### Table 6.1: Parameter values for the female supply model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_f$</td>
<td>Birth/death rate</td>
<td>$0 \frac{0.025}{365}$</td>
</tr>
<tr>
<td>$\beta_{VP}$</td>
<td>Recruitment rate of voluntary prostitutes (initial value)</td>
<td>$1 \frac{1}{365}$</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>scaling factor for voluntary prostitutes ($\frac{\text{Voluntary prostitutes}}{\text{transactions}}$)</td>
<td>$0.1 \frac{0.1}{365}$</td>
</tr>
<tr>
<td>$\beta_{TP}$</td>
<td>Recruitment rate of trafficked prostitutes (initial value)</td>
<td>$2 \frac{2}{365}$</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>scaling factor for trafficked prostitutes ($\frac{\text{Trafficked prostitutes}}{\text{transactions}}$)</td>
<td>$0.2 \frac{0.2}{365}$</td>
</tr>
<tr>
<td>$k_m$</td>
<td>Maximum sexual activity desired by an average customer</td>
<td>$50 \frac{50}{365}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Relapse rate of rehabilitated commercial sex workers (initial value)</td>
<td>$3 \frac{3}{365}$</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>scaling factor ($\frac{\text{Prostitutes}}{\text{transactions}}$)</td>
<td>$0.3 \frac{0.3}{365}$</td>
</tr>
<tr>
<td>$\gamma_f$</td>
<td>Arrest rate of commercial sex workers</td>
<td>$0.0137 \frac{0.0137}{365}$</td>
</tr>
<tr>
<td>$\kappa_f$</td>
<td>Rate of leaving jail</td>
<td>$0.15 \frac{0.15}{365}$</td>
</tr>
<tr>
<td>$\delta_f$</td>
<td>Proportion of jailed workers who immediately resume working</td>
<td>$0.5$</td>
</tr>
</tbody>
</table>
Figures 6.3 and 6.4 show the results of the male model.

As we can see from these figures, the male model exhibited very similar behaviour to our modified model in section 5.5. The growth of all the populations is slow and the steady state values are also almost the same as the modified values, implying that our model works efficiently. The active customers stabilize at a value of 51%, the potential customers stabilize at 26.5%, the abstainers at 21% and finally, the jailed customers at 0.85%. This also confirms the fact that the supply side of prostitution does not play a major role in determining the demand for prostitution services.
The female trafficking model is also seen to exhibit behaviour similar to the modified model in section 5.5. Since we have stratified the population of prostitutes to include human trafficking, the values of the populations of prostitutes are slightly higher than the population of prostitutes in our previous model. The results of the female model are graphically shown in Figure 6.5. To meet the male demand, voluntary prostitutes stabilize at approximately 9% and trafficked prostitutes stabilize at 15.5%. Jailed workers constitute about 2% of the population and rehabilitated workers constitute 4%. Finally, the population of susceptible women stays steady at approximately 68.6%. There is about a 6 percentage point increase in prostitution as compared to our previous model. The exact steady state values for each model are
given in Table 6.2. We now use these results as a foundation for comparison and move onto analyzing other types of regulation relating to prostitution.

![Figure 6.5: Female model under illegal prostitution](image)
### 6. Human Trafficking Model

<table>
<thead>
<tr>
<th>Population</th>
<th>Initial value</th>
<th>Steady state value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Susceptible females</td>
<td>0.8</td>
<td>0.6866</td>
</tr>
<tr>
<td>Voluntary prostitutes</td>
<td>0.05</td>
<td>0.0939</td>
</tr>
<tr>
<td>Trafficked prostitutes</td>
<td>0.1</td>
<td>0.1555</td>
</tr>
<tr>
<td>Jailed prostitutes</td>
<td>0.02</td>
<td>0.0195</td>
</tr>
<tr>
<td>Rehabilitated prostitutes</td>
<td>0.03</td>
<td>0.04450</td>
</tr>
<tr>
<td>Potential customers</td>
<td>0.55</td>
<td>0.2653</td>
</tr>
<tr>
<td>Abstainers</td>
<td>0.1</td>
<td>0.2131</td>
</tr>
<tr>
<td>Active customers</td>
<td>0.349</td>
<td>0.5130</td>
</tr>
<tr>
<td>Jailed customers</td>
<td>0.001</td>
<td>0.0085</td>
</tr>
</tbody>
</table>

**Table 6.2:** Different values of each population at steady state under illegal prostitution.

### 6.2.2 Legal Prostitution: Netherlands

We now analyze a situation in which all forms of prostitution are legal. The Netherlands is a good example of such a situation since it legalized all forms of prostitution on October 1, 2000. Street prostitution, brothel ownership and ‘pimping’ are all legal forms of prostitution found rampantly in the country [6]. This legislation was passed in the hopes of giving the government more power in controlling the working conditions of women in prostitution and regulating the industry in order to curb trafficking. But according to the Dutch government, it has not been able to draw any significant conclusions about the impact of the law [6] [29]. Due to the legalization of prostitution, the demand for prostitution services has significantly increased, with one study claiming that the sex industry increased by 25% in the Netherlands and accounted for 5% of the economy [20] [23]. On the other hand, women in the Netherlands also point out that even though legalized, the stigma of prostitution has not decreased [23]. Thus, there is an extremely dwindling amount
of Dutch women voluntarily involved in prostitution. Furthermore, studies find that about 60-80% of prostitutes in the Netherlands originate from other countries such as Albania, Romania and Nigeria [29], with the majority of them being trafficked, [24] [29] due to the demand for a variety of ‘bodies’ proving to be an incentive for traffickers [22]. Another incentive for brothel owners to traffick victims stems from the fact that legalization allows them to increase prices and earn more profits by exploiting slave labour. Legalization has thus reduced the risk involved in trafficking, and even though regular inspections are supposed to be carried out, bribes to the local mayor cause such inspections to be a rare occasion [29].

Even though some experts indicate that trafficking has increased since legalization, there is inconclusive data to fully support the argument [29]. Thus, we now incorporate the aspects of complete legalization in our model to determine the effect of such regulation. Legalization affects many aspects of our model. To reflect the increase in demand, the recruitment rate of potential to active customers, \( \eta_1 \), must increase. To do so, we have increased \( \eta_m \), the maximum recruitment of active customers. We have also increased \( k_m \), the maximum sexual activity desired by the average male customer, as it is also affected by an increase in demand. We have also slightly decreased the value of \( \eta_2 \), the recruitment rate of abstainers, since we assume that the increase in demand is also through customers who were initially abstainers before legalization as the reduced risk now provides them with a reasonable incentive to indulge in such services. Since there are almost no customer arrests and prostitute arrests, we have significantly decreased the values of the active customer arrest rate, \( \gamma_m \) and the arrest rate for prostitutes, \( \gamma_f \). It is also assumed that the proportion of the active customers who temporarily ceased activity due to a fear of being arrested do not cease their activity anymore. Thus, we have also reduced, \( \omega_0 \) which affects the rate of ceasing activity due to the knowledge of arrests. Lastly, even though there is still a major stigma associated with the industry, we have
slightly increased the value of $\beta_1$ which affects the recruitment rate of voluntary prostitutes to allow for individuals to choose to enter the industry voluntarily. We do not yet change the value of the recruitment rate of trafficked victims in order to see how the changes in the other parameters affect the population of trafficked victims. These changes are listed in Tables 6.3 and 6.4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_f$</td>
<td>$\frac{0.025}{365}$</td>
<td>$\mu_m$</td>
<td>$\frac{0.025}{365}$</td>
</tr>
<tr>
<td>$\beta_{VP}$ (initial)</td>
<td>$\frac{1}{365}$</td>
<td>$\Lambda$</td>
<td>0.9</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>$\frac{0.15}{365}$</td>
<td>$\eta_1$ (initial)</td>
<td>$\frac{0.5}{365}$</td>
</tr>
<tr>
<td>$\beta_{TP}$ (initial)</td>
<td>$\frac{2}{365}$</td>
<td>$\eta_m$</td>
<td>$\frac{0.6}{365}$</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>$\frac{0.2}{365}$</td>
<td>$\epsilon$</td>
<td>0.001</td>
</tr>
<tr>
<td>$k_m$</td>
<td>$\frac{60}{365}$</td>
<td>$\eta_2$</td>
<td>$\frac{0.03}{365}$</td>
</tr>
<tr>
<td>$\rho$ (initial)</td>
<td>$\frac{3}{365}$</td>
<td>$\gamma_m$</td>
<td>$\frac{0.025}{365}$</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>$\frac{0.3}{365}$</td>
<td>$\kappa_m$</td>
<td>$\frac{15}{365}$</td>
</tr>
<tr>
<td>$\gamma_f$</td>
<td>$\frac{0.00137}{365}$</td>
<td>$\delta_m$</td>
<td>0.7</td>
</tr>
<tr>
<td>$\kappa_f$</td>
<td>$\frac{0.15}{365}$</td>
<td>$\omega$ (initial)</td>
<td>$\frac{0.5}{365}$</td>
</tr>
<tr>
<td>$\delta_f$</td>
<td>0.5</td>
<td>$\omega_0$</td>
<td>$\frac{0.3}{365}$</td>
</tr>
</tbody>
</table>

**Table 6.3:** Parameter values for the female supply model under legalization.

**Table 6.4:** Parameter values for the male demand model under legalization.
The results of the male model are shown graphically in Figures 6.6 and 6.7.

The male populations seem to grow at a much faster rate under complete legalization than under illegal prostitution and are close to a steady state after about 5-8 years. Under a system of legal prostitution, the population of active customers rises to a population size of approximately 84%, which is a huge increase from the population under illegal prostitution (51%). The population of potential customers sees a major decline at 5.5% of the population, while the populations of abstainers and jailed customers also see a decrease at 10% and 0.1% respectively. Thus, we find that legalization has a tremendous impact on the demand for prostitution services, with the active customers reaching an exorbitant proportion of the population. This
result mostly stems from the fact that legalization completely diminishes the risk faced by customers of commercial sex. Furthermore, with no fear of arrests, it seems to provide an incentive to abstainers to be recruited into the class of active customers. This may also be attributed to the fact that indulging in prostitution may be considered less of a moral evil under legalization, and even though the stigma for prostitution remains, the ostracism from society usually falls on the women involved in the prostitution industry rather than the men involved. Thus, individuals who were initially abstainers for reasons other than the risk involved, may find new incentives to indulge in these services, thus decreasing the proportion of abstainers in the population.

Figure 6.7: Male model under legal prostitution (68,000 days)
6. Human Trafficking Model

The female populations on the other hand grow at a slower rate, taking a much longer time to reach a steady state than the male model. There is a big difference in the results of the model when prostitution is illegal versus when prostitution is legal. The population of trafficked victims do seem to increase significantly from about 16% under illegal prostitution to 45% under legal prostitution. The population of voluntary prostitutes sees a decrease and stabilizes at 5%. Finally, the population of susceptibles, jailed workers and rehabilitated workers also see a decrease and stabilize at 48%, 0.4% and 1% respectively. Therefore, we find that legalizing prostitution has drastic effects on the population of trafficked victims, with almost the majority of the population entering the trafficking industry. This is mostly due to the fact that legalization significantly increases the demand for prostitution. Thus, traffickers and brothel owners may find the need to increase the number of trafficked prostitutes employed. In contrast, interestingly, we find that the population of voluntary prostitutes sees a decrease despite the recruitment rate of voluntary prostitutes being increased as well as the risk of being a prostitute represented by the arrest rate of prostitutes being decreased. A part of this may be attributed to the fact that other women may not want to enter this industry on hearing that the emotional well-being of prostitutes is found to have further decreased since the legalization of prostitution [19]. However, exploring this result of the model may provide a good platform for future work.

These results are graphically shown in Figure 6.8. The exact numerical results of the models are given in Table 6.5.
6. Human Trafficking Model

Figure 6.8: Female model under legal prostitution (502,000 days)

<table>
<thead>
<tr>
<th>Population</th>
<th>Initial value</th>
<th>Steady state value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Susceptible females</td>
<td>0.8</td>
<td>0.4826</td>
</tr>
<tr>
<td>Voluntary prostitutes</td>
<td>0.05</td>
<td>0.0513</td>
</tr>
<tr>
<td>Trafficked prostitutes</td>
<td>0.1</td>
<td>0.4520</td>
</tr>
<tr>
<td>Jailed prostitutes</td>
<td>0.02</td>
<td>0.0039</td>
</tr>
<tr>
<td>Rehabilitated prostitutes</td>
<td>0.03</td>
<td>0.0101</td>
</tr>
<tr>
<td>Potential customers</td>
<td>0.55</td>
<td>0.0551</td>
</tr>
<tr>
<td>Abstainers</td>
<td>0.1</td>
<td>0.1071</td>
</tr>
<tr>
<td>Active customers</td>
<td>0.349</td>
<td>0.8364</td>
</tr>
<tr>
<td>Jailed customers</td>
<td>0.001</td>
<td>0.0014</td>
</tr>
</tbody>
</table>

Table 6.5: Different values of each population at steady state under legal prostitution.
These results were produced under the assumption that the recruitment rate for trafficked prostitutes remains unchanged. But in reality, the rate of recruitment for trafficked prostitutes does see an increase due to multiple reasons. First, the risk faced by traffickers is dramatically lowered. A large chunk of traffickers in the Netherlands further reduce their risk as they coach their victims to describe themselves as ‘independent migrant sex workers,’ thereby deceiving legal authorities [23]. Second, like mentioned earlier, brothel owners have a lot of financial incentive to increase the number of trafficked prostitutes employed. Higher prices and minimal labour costs force this recruitment rate to increase. Thus, in our model, we increased the value of $\beta_2$, which proportionally affects the recruitment rate of trafficked prostitutes, very slightly to see the effect it would have on the populations. While the results of the male model were the same (since those populations are unaffected as customers usually do not usually know whether an individual is trafficked or is voluntarily employed in the profession [29]), the population of trafficked prostitutes further increased from 45% to a staggering 52%, almost completely wiping out the population of voluntary prostitutes. Even though such a result is an exorbitant estimate of the actual population sizes, it goes to show that trafficking plays a much bigger role than voluntary entrance, in the prostitution industry. Thus, under legal prostitution, human trafficking is seen to have a very significant increase as is seen in the case of the Netherlands. We now analyze a situation in which prostitution has limited legality.

### 6.2.3 Limited Legality: Sweden

We now analyze the effects of limited legality on the market for prostitution by looking at the workings of the industry in Sweden. Sweden has a very interesting form of regulation surrounding prostitution. It decriminalizes the women involved in prostitution but criminalizes the buyers of prostitution services. In other words,
6. Human Trafficking Model

it is legal to work as a prostitute in Sweden, but it is illegal to purchase sexual services. This law was enacted on January 1, 1999 and was the first law introduce by a country that focused on reducing the demand for sexual services. The argument given was that if the demand for sexual services can be significantly decreased, then the number of women being trafficked for this purpose would in turn see a significant decline [38] [8]. Although there is not sufficient evidence to support the claim that prostitution has been significantly reduced, it has also not increased since the ban on buying sexual services has been enacted [8] [30]. At the same time, some researchers say that the criminalization of buyers has led to the sex industry going further underground, giving rise to internet and indoor prostitution, which is difficult to track [8] [47]. Furthermore, male customer arrests have seen a significant increase since the implementation of the ban [30].

We now implement these conditions in our model to see its affect on trafficking. Since the regulation criminilizes the buyers of commercial sex, we significantly increase the male arrest rate, $\gamma_m$. On the other hand, since the regulation decriminalizes the prostitutes involved in the business, we decrease the female arrest rate, $\gamma_f$. We also increase $\omega_0$, which affects the rate of ceasing activity due to the knowledge of arrests, since it is now assumed that more customers would stop indulging in prostitution services after hearing about significant increases in arrests. The new values of the parameters that have changed for the male model are shown in Table 6.6 below. (The other values remain the same as those under illegal prostitution as we assume that they do not get affected.)
6. Human Trafficking Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_m$</td>
<td>$\frac{0.4}{365}$</td>
</tr>
<tr>
<td>$\gamma_f$</td>
<td>$\frac{0.00137}{365}$</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>$\frac{0.6}{365}$</td>
</tr>
</tbody>
</table>

Table 6.6: Parameter values that change under limited legalization.

The results of this model are extremely different than the results of any of the previous models. We find that prostitution in general significantly decreases. For the female model, the population of susceptibles stabilize at 82%, voluntary prostitutes at 0.7%, trafficked prostitutes at 16%, jailed workers at 0.1% and rehabilitated workers at 0.4%. The total number of prostitutes has thus significantly decreased, although about 95% of those prostitutes seem to be trafficked according to our results. This may be because women may not see a viable financial incentive to voluntarily enter the prostitution industry since the demand for prostitution has dramatically decreased. On the other hand, due to the strict regulations imposed on prostitution, most of the business is assumed to have been pushed further underground. Thus, to fulfill the demand that remains, brothel owners may be induced to employ trafficked victims as they represent lower labour costs. Another observation to make note of in this model is that the ratio of active customers to prostitutes has decreased. The Swedish model of limited legality in the prostitution industry, thus, seems to have the most positively impactful results on the market for human trafficking. Even though the majority of prostitutes in the industry are trafficked, the proportion of
the population representing trafficked victims is significantly lower than those of the previous models. These results are graphically shown in Figure 6.9.

For the male mode, the population of potential customers settles at 38%, abstainers at 42%, active customers at 20% and jailed customers at 0.5%. Once again, the male model changes much faster than the female model under limited legalization. Figure 6.10 shows that the criminalization of customers has significantly impacted the demand for prostitution services, thus forcing more and more consumers to refrain from purchasing such services. Interestingly, the population of jailed customers stays low at 0.5% of the population in contrast to the belief that a higher arrest rate would lead to a higher population of jailed customers. Thus, more consumers are
found to permanently refrain from such activity than indulge in such services with a higher risk of being prosecuted.

![Figure 6.10: Male model under limited legalization (600,000 days)](image_url)

Another interesting observation to be made in this model is that given these parameters, if we reduce $\delta_m$, the proportion of detained customers who immediately resume activity (currently at 0.7), to a proportion below 0.67, or if we reduce $\eta_m$, the maximum recruitment rate of potential to active customers (currently at 0.4), to a rate of 0.39 or below, we find that there is no demand for prostitution in the market. The active and jailed customers are both 0, as the risk involved may be deemed to be much higher than the utility derived from buying such a service, and potential customers and abstainers together make 100% of the population. As a result, in the female model, there is no supply of prostitutes, and thus, susceptible women make
100% of the population. However, such a result is unlikely to happen, as research suggests that regardless of the regulations involved, the industry would just move deeper underground.

However, we find that if we reduce the recruitment rate of trafficked victims, the population of trafficked victims sees a significant decrease, forcing the population of voluntary prostitutes to increase to fulfill the existing demand of the customers. To do this, we reduced the value of $\beta_2$ (originally $\frac{0.2}{365}$), which directly affects the recruitment rate of trafficked prostitutes, and the value of $\beta_1$ (originally $\frac{0.1}{365}$), which directly affects the recruitment rate of voluntary prostitutes, to make them both equal to $\frac{0.15}{365}$. The results of the simulation showed that although there was no significant change in the male model, with the population of active customers still approximately 20%, the population of trafficked victims sharply declined to a population of 2% whereas the population of voluntary prostitutes was forced to increase to a population of approximately 11% to meet the demand. In addition, we found that if we further reduce the recruitment rate of trafficked prostitutes such that it is less than the recruitment rate of voluntary prostitutes (therefore, $\beta_2 = \frac{0.1}{365} < \beta_1 = \frac{0.2}{365}$), we find that the population of trafficked victims is completely removed from the system at 0%, whereas the population of voluntary prostitutes rises to 17%. Once again, there is no change in the male populations. This result seems logical as it implies that if the recruitment rate for trafficked victims is lower than recruitment rate of voluntary prostitutes, thus if it is easier to employ a voluntary worker as compared to a trafficked worker, trafficking in the prostitution industry would cease to exist. Traffickers and brothel owners would lose the incentive to find and traffic women into the industry. Instead, brothel owners would only employ voluntary workers as that would represent a more cost-efficient option.
6. Human Trafficking Model

The numerical results of the male and female models are shown in Table 6.7 below.

<table>
<thead>
<tr>
<th>Population</th>
<th>Initial value</th>
<th>Steady state value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Susceptible females</td>
<td>0.8</td>
<td>0.8235</td>
</tr>
<tr>
<td>Voluntary prostitutes</td>
<td>0.05</td>
<td>0.0077</td>
</tr>
<tr>
<td>Trafficked prostitutes</td>
<td>0.1</td>
<td>0.1635</td>
</tr>
<tr>
<td>Jailed prostitutes</td>
<td>0.02</td>
<td>0.0013</td>
</tr>
<tr>
<td>Rehabilitated prostitutes</td>
<td>0.03</td>
<td>0.004</td>
</tr>
<tr>
<td>Potential customers</td>
<td>0.55</td>
<td>0.3795</td>
</tr>
<tr>
<td>Abstainers</td>
<td>0.1</td>
<td>0.4150</td>
</tr>
<tr>
<td>Active customers</td>
<td>0.349</td>
<td>0.2001</td>
</tr>
<tr>
<td>Jailed customers</td>
<td>0.001</td>
<td>0.0053</td>
</tr>
</tbody>
</table>

Table 6.7: Values of each population at steady state under limited legalization.

Thus, in both the cases of regulation relating to legal prostitution, we have observed that human trafficking is strongly influenced by the legality of prostitution. Even though the supply of prostitutes does not seem to have much of an effect on the demand for prostitution services, an increase in the demand for prostitution causes significant increases in the population of prostitutes. Both ends of the spectrum of legal prostitution, complete legalization and limited legalization, show that trafficked prostitutes greatly outnumber voluntary prostitutes in terms of fulfilling the level of demand observed for such services. Thus, we can conclude that the legalization of prostitution affects the market for human trafficking by increasing the level of trafficked victims in the industry, especially in the case of absolute legalization.
CHAPTER 7

CONCLUSION

This research has attempted to investigate the relationship between the legalization of prostitution and the market for human trafficking. We began our study by introducing the reader to the world of prostitution and human trafficking, followed by a brief survey of the literature. We then explored the concepts of a compartmental epidemiological model and utilized that knowledge to expand on the model presented by Davidoff et al. (2006) [21] and create a realistic base model that included the element of human trafficking. We then used our model to perform numerical simulations to analyze the effects of legalization on trafficked individuals. An increase in the level of trafficked victims employed in the prostitution industry was anticipated as a result of legalized prostitution. This was confirmed by the numerical simulations carried out in Chapter 6.

We found that under complete legalization, there was a huge rise in the populations of both the active customers and trafficked prostitutes. The reduction of the level of risk involved in indulging in prostitution services increased the active customer population from about 51% under a system of illegal prostitution, to approximately 84% of the entire population under complete legalization. This steep increase in demand cause trafficked prostitutes to rise from a population of about 15%, under a system of illegal prostitution, to a population of about 45% under legalization. This was attributed to the fact that traffickers and brothel owners faced a very
low intensity of risk and a high margin of profit for employing trafficked workers through low labour costs due to the exploitation of trafficked victims through the debt-bondage system described in Chapter 2. In contrast, the population of voluntary prostitutes saw a decrease to about 5% of the population. Possible reasons for this decrease include a continued stigma attached to the world of prostitution, with a reinforcement of the objectification of women due to legalization, high levels of emotional abuse and a fear of ostracism that may have deterred more women from entering the industry.

Under the Swedish system of limited legality, we found that prostitution levels significantly reduced to a total of 16.7% due to a high reduction in the demand for such services from the male model. Although, trafficked workers represented 95% of the population of total prostitutes in this model. Thus, trafficked prostitutes were once again seen to be a bigger component of the supply than voluntary prostitutes.

A key takeaway from this study is that the demand for prostitution services has a major impact on the supply of prostitutes, although the reverse is not seen to be true. Thus, reducing the demand for such services through stricter regulation imposed on customers, as seen in the Swedish model, goes a long way in reducing the level of prostitutes in the industry. However, this is not enough to significantly curb the volume of trafficking involved, also seen in the Swedish model. Trafficking is found to reduce only if the demand for commercial sex is completely removed. As long as even a certain amount of demand remains, the number of women trafficked into this industry will always outnumber the number of women voluntarily working in the industry, given the current forms of entry and exit. Efforts should thus focus on reducing the recruitment rate of active customers into the system, $\eta_1$. This would eventually help eradicate the demand for such services and thereby help in the significant reduction of trafficking. In addition, efforts by legal authorities should
also focus on creating barriers to entry for trafficked victims into the prostitution industry. This would affect the labour costs faced by traffickers and brothel owners, and would significantly reduce the incentive to employ trafficked victims over voluntary workers. Therefore legal efforts should also focus on reducing $\beta_2$, the recruitment rate of trafficked victims.

Though this study has successfully established a relationship between the legalization of prostitution and the market for human trafficking, there are many limitations that need to be addressed in the future work on the subject.

7.0.1 Limitations and Future Work

The trafficking model developed in this research has a number of limitations. First, it assumes the same birth and death rate for both the male and female models. Second, the structure of the system does not allow for the modeling of underground prostitution, such as indoor and internet prostitution. Third, it does not allow for an inflow of individuals from the jailed and rehabilitated populations back into the trafficking population. These concerns should be addressed in all future work done using this model.

In addition, now that we have established a working base model, there is tremendous opportunity, that we have not had the time to pursue in this research, for an in depth analysis of country specific models under the same type of regulations. Different combinations of parameter changes, like what was shown at the very end of the analysis for the Swedish model in Chapter 6, should also be analyzed in further detail. There is also ample opportunity to strengthen the basic model to better reflect the theory and statistics found in the literature. This will provide a stronger base for the results and conclusions of the trafficking model, and will allow us to make better recommendations for the reduction of human trafficking in the future.
This appendix lists the MATLAB code used to simulate the individual model.

Listing A.1: Single Model

```matlab
clear all;
close all;

iterations = 1500;

%female model parameters
beta = 1/365;
mu = 1/14600;
gamma_f = 1.37/365;
delta_f = 0.5;
kappa_f = 15/365;
rho = 3/365;

%male model parameters
lambda = 0.9;
eta_1 = 30/365;
etta_2 = 0.25/365;
gamma_m = 0.25/365;
omega = 0.5/365;
kappa_m = 15/365;
delta_m = 0.7;

%female model population vectors
S = zeros(1,iterations);
P = zeros(1,iterations);
J_f = zeros(1,iterations);
R = zeros(1,iterations);
```
A. Single Model

%male model population vectors
C_p = zeros(1, iterations);
G = zeros(1, iterations);
C_a = zeros(1, iterations);
J_m = zeros(1, iterations);

%female model initial values
S(1,1) = 0.8;
P(1,1) = 0.1;
J_f(1,1) = 0.05;
R(1,1) = 0.05;
N = 1.0; %Total population

%male model initial values
C_p(1,1) = 0.8;
G(1,1) = 0.1;
C_a(1,1) = 0.05;
J_m(1,1) = 0.05;
M = 1.0; %Total population

for t=2:1:iterations
S(1,t) = S(1,t-1) + mu*N - beta*((S(1,t-1)*P(1,t-1))/N) - mu*(S(1,t-1));
P(1,t) = P(1,t-1) + beta*((S(1,t-1)*P(1,t-1))/N) - (gamma_f + mu)*(P(1,t-1)) + (delta_f*kappa_f*(J_f(1,t-1)))+rho*((R(1,t-1)*P(1,t-1))/N);
J_f(1,t) = J_f(1,t-1) + gamma_f*P(1,t-1) - (kappa_f + mu)*(J_f(1,t-1));
R(1,t) = R(1,t-1) + (1-delta_f)*kappa_f*(J_f(1,t-1)) - rho*((R(1,t-1))*(P(1,t-1))/N) - mu*(R(1,t-1));
C_p(1,t) = C_p(1,t-1) + lambda*mu*M - ((eta_1*(C_p(1,t-1))*(C_a(1,t-1)))/M) - ((eta_2*(C_p(1,t-1))*(G(1,t-1)))/M) + (omega*(C_a(1,t-1))) + ((1-delta_m)*kappa_m*(J_m(1,t-1))) - (mu*(C_p(1,t-1)));
G(1,t) = G(1,t-1) + (1-lambda)*mu*M + ((eta_2*(C_p(1,t-1))*(G(1,t-1)))/M) - mu*(G(1,t-1));
C_a(1,t) = C_a(1,t-1) + ((eta_1*(C_p(1,t-1))*(C_a(1,t-1)))/M) + delta_m*kappa_m*(J_m(1,t-1)) - ((gamma_m + omega + mu)*(C_a(1,t-1)));
J_m(1,t) = J_m(1,t-1) + (gamma_m*(C_a(1,t-1))) - ((kappa_m + mu)*(J_m(1,t-1)));
A. Single Model

end

x = 1:iterations;

% storing female model equilibrium populations

equilibrium_f = 4:1;
equilibrium_f(1,1) = S(1,iterations);
equilibrium_f(2,1) = P(1,iterations);
equilibrium_f(3,1) = J_f(1,iterations);
equilibrium_f(4,1) = R(1,iterations);
equilibrium_f % displaying female equilibriums

% storing male model equilibrium populations

equilibrium_m = 4:1;
equilibrium_m(1,1) = C_p(1,iterations);
equilibrium_m(2,1) = G(1,iterations);
equilibrium_m(3,1) = C_a(1,iterations);
equilibrium_m(4,1) = J_m(1,iterations);
equilibrium_m % displaying male equilibriums

% plotting the female populations against time
plot(x,S,'k*',x,P,'r--',x,J_f,'g+',x,R,'b^');
axis([0 iterations 0 1]);
xlabel('Days');
ylabel('Populations across time');
legend('Susceptible','Prostitute','Jailed','Rehabilitated');
figure;

% plotting the male populations against time
plot(x,C_p,'k*',x,G,'go',x,C_a,'r--',x,J_m,'b^');
axis([0 iterations 0 1]);
xlabel('Days');
ylabel('Populations across time');
legend('Potential_Customer','Abstainer','Active_Customer','Jailed');

R_m = ((eta_1*C_p(1,1))/(omega + gamma_m + mu)) + delta_m *
     ((gamma_m)/(omega + gamma_m + mu)) *((kappa_m)/(mu + kappa_m))'
This appendix lists the MATLAB code used to simulate the coupled model.

```
Listing B.1: Coupled Model

1 clc all;
2 close all;
3
4 iterations = 11000;
5
6 %female model parameters
7  %beta = 1/365;
8  mu = 1/14600;
9  %gamma_f = 1.37/365;
10  delta_f = 0.5;
11  kappa_f = 15/365;
12  %rho = 3/365;
13
14 %male model parameters
15  lambda = 0.9;
16  %eta_1 = 30/365;
17  eta_2 = 0.25/365;
18  %gamma_m = 0.25/365;
19  %omega = 0.5/365;
20  kappa_m = 15/365;
21  delta_m = 0.7;
22
23 %variable constants
24  beta_0 = 1/365;
25  k_m = 50/365;
26  rho_0 = 3/365;
27  eta_m = 50/365;
28  epsilon = 0.001;
```
B. Coupled Model

\begin{verbatim}
    sigma_m = 0.3/365;
    sigma_f = 1.5/365;
    omega_0 = 0.5/365;

    %variable vectors
    beta = zeros(1,iterations);
    rho = zeros(1,iterations);
    eta_1 = zeros(1,iterations);
    gamma_m = zeros(1,iterations);
    gamma_f = zeros(1,iterations);
    omega = zeros(1,iterations);

    %female model population vectors
    S = zeros(1,iterations);
    P = zeros(1,iterations);
    J_f = zeros(1,iterations);
    R = zeros(1,iterations);

    %male model population vectors
    C_p = zeros(1,iterations);
    G = zeros(1,iterations);
    C_a = zeros(1,iterations);
    J_m = zeros(1,iterations);

    %female model initial values
    N = 1; %Total population
    S(1,1) = 0.8;
    P(1,1) = 0.1;
    J_f(1,1) = 0.05;
    R(1,1) = 0.05;

    %male model initial values
    M = 1; %Total population
    C_p(1,1) = 0.8;
    G(1,1) = 0.1;
    C_a(1,1) = 0.05;
    J_m(1,1) = 0.05;

    %variable initial values
    beta(1,1) = 1/365;
    rho(1,1) = 3/365;
    eta_1(1,1) = 30/365;
    gamma_m(1,1) = 0.25/365;
    gamma_f(1,1) = 1.37/365;
    omega(1,1) = 0.5/365;
\end{verbatim}
B. Coupled Model

%female model individual components

birth_f = 1:iterations;
flow_SP=1:iterations;
death_S= 1:iterations;
flow_PJ=1:iterations;
death_P=1:iterations;
flow_JP=1:iterations;
flow_RP=1:iterations;
death_J_f=1:iterations;
flow_JR=1:iterations;
death_R=1:iterations;
kapJ_f=1:iterations;

%male model indivdual components

birth_m = 1:iterations;
flow_Cp_Ca = 1:iterations;
flow_Ca_Cp = 1:iterations;
flow_J_Cp = 1:iterations;
death_Cp = 1:iterations;
flow_J_Ca = 1:iterations;
flow_Ca_J = 1:iterations;
death_Ca = 1:iterations;
kapJ_m=1:iterations;
death_J_m = 1:iterations;
test = 1:iterations;

for t=2:1:iterations;

%running the female model

S(1,t) = S(1,t-1) +mu*N - (beta(1,t-1))*((S(1,t-1)*P(1,t-1)/N) -mu*(S(1,t-1));
P(1,t) = P(1,t-1) + (beta(1,t-1)) *((S(1,t-1)*P(1,t-1)/N) -(gamma_f(1,t-1) + mu)*(P(1,t-1)) + (delta_f*kappa_f*(J_f(1,t-1)))+(rho(1,t-1))*((R(1,t-1)*P(1,t-1))/N);
J_f(1,t) = J_f(1,t-1) + (gamma_f(1,t-1))*P(1,t-1) -(kappa_f + mu)*(J_f(1,t-1));
R(1,t) = R(1,t-1) + (1-delta_f)*kappa_f*(J_f(1,t-1)) -(rho(1,t-1))*((R(1,t-1))*(P(1,t-1))/N) -mu*(R(1,t-1));

%running the male model
\[ C_p(1, t) = C_p(1, t-1) + \lambda * \mu * M - ((\eta_1(1, t-1)) * (C_p(1, t-1)) * (C_a(1, t-1)) / M) - ((\eta_2(1, t-1)) * (C_p(1, t-1)) * (G(1, t-1)) / M) + ((\omega(1, t-1)) * (C_a(1, t-1)) + ((1-\delta_m) * \kappa_m * (J_m(1, t-1))) - (\mu * (C_p(1, t-1))); \]
\[ G(1, t) = G(1, t-1) + (1-\lambda * \mu * M + ((\eta_2(1, t-1)) * (C_p(1, t-1)) * (G(1, t-1))) / M) - \mu * (G(1, t-1)); \]
\[ C_a(1, t) = C_a(1, t-1) + ((\eta_1(1, t-1)) * (C_p(1, t-1)) * (C_a(1, t-1)) / M) + (\delta_m * \kappa_m * (J_m(1, t-1))) - ((\gamma_m(1, t-1)) + (\omega(1, t-1)) + (\mu)) * (C_a(1, t-1)); \]
\[ J_m(1, t) = J_m(1, t-1) + ((\gamma_m(1, t-1)) * (C_a(1, t-1))) - ((\kappa_m * \mu * (J_m(1, t-1))); \]

%running the combined parametres
\[ \beta(1, t) = \beta_0 * \kappa_m * ((C_a(1, t-1)) / P(1, t-1)); \]
\[ \rho(1, t) = \rho_0 * \kappa_m * ((C_a(1, t-1)) / P(1, t-1)); \]
\[ \eta_1(1, t) = ((\eta_2(1, t-1)) / ((P(1, t-1)) + \varepsilon * N)); \]
\[ \gamma_m(1, t) = (\sigma_m * (C_a(1, t-1)) / M); \]
\[ \gamma_f(1, t) = (\sigma_f * (P(1, t-1)) / N); \]
\[ \omega(1, t) = ((\eta_0 * (J_m(1, t-1))) / ((C_a(1, t-1)) + (C_p(1, t-1)) + (J_m(1, t-1)))); \]

%running the female indivudal components
\[ \text{birth}_f(1, t) = \mu * N; \]
\[ \text{flow}_SP(1, t) = (\beta(1, t-1)) * (S(1, t-1) * P(1, t-1)) / N; \]
\[ \text{death}_S(1, t) = \mu * (S(1, t-1)); \]
\[ \text{flow}_PJ(1, t) = (\gamma_m(1, t-1)) * P(1, t-1); \]
\[ \text{death}_P(1, t) = (\mu) * (P(1, t-1)); \]
\[ \text{flow}_JP(1, t) = (\delta_f * \kappa_f * (J_f(1, t-1))); \]
\[ \text{flow}_RP(1, t) = (\rho(1, t-1)) * ((R(1, t-1) * P(1, t-1)) / N); \]
\[ \text{death}_J_f(1, t) = \mu * (J_f(1, t-1)); \]
\[ \text{flow}_JR(1, t) = (1-\delta_f) * \kappa_f * (J_f(1, t-1)); \]
\[ \text{death}_R(1, t) = \mu * (R(1, t-1)); \]
\[ \text{kap}J_f(1, t) = \kappa_f * (J_f(1, t-1)); \]

%running the female indivudal components
\[ \text{birth}_m(1, t) = \lambda * \mu * M; \]
\[ \text{flow}_CP_Ca(1, t) = ((\eta_1(1, t-1)) * (C_p(1, t-1)) * (C_a(1, t-1)) / M); \]
\[ \text{flow}_Ca_CP(1, t) = ((\omega(1, t-1)) * (C_a(1, t-1))); \]
\[ \text{flow}_I_Cp(1, t) = ((1-\delta_m) * \kappa_m * (J_m(1, t-1))); \]
\[ \text{death}_Cp(1, t) = (\mu * (C_p(1, t-1))); \]
\[ \text{flow}_Ca_Ca(1, t) = \delta_m * \kappa_m * (J_m(1, t-1)); \]
\[ \text{flow}_Ca_J(1, t) = (\gamma_m(1, t-1)) * (C_a(1, t-1)); \]
\[ \text{death}_Ca(1, t) = \mu * (C_a(1, t-1)); \]
B. Coupled Model

\[
\text{kap}_J_m(1,t) = \kappa_m(J_m(1,t-1));
\]
\[
\text{death}_J_m(1,t) = \mu(J_m(1,t-1));
\]
\[
\text{test}(1,t) = ((C_p(1,t-1)) + C_a(1,t-1) + (J_m(1,t-1)) + G(1,t-1));
\]
\[
\text{end}
\]
\[
x = 1: \text{iterations};
\]
\[
\% \text{storing female model equilibrium populations}
\]
\[
equilibrium_f = 4:1;
\]
\[
equilibrium_f(1,1) = S(1,\text{iterations});
\]
\[
equilibrium_f(2,1) = P(1,\text{iterations});
\]
\[
equilibrium_f(3,1) = J_f(1,\text{iterations});
\]
\[
equilibrium_f(4,1) = R(1,\text{iterations});
\]
\[
equilibrium_f \% \text{displaying female equilibriums}
\]
\[
\% \text{storing male model equilibrium populations}
\]
\[
equilibrium_m = 4:1;
\]
\[
equilibrium_m(1,1) = C_p(1,\text{iterations});
\]
\[
equilibrium_m(2,1) = G(1,\text{iterations});
\]
\[
equilibrium_m(3,1) = C_a(1,\text{iterations});
\]
\[
equilibrium_m(4,1) = J_m(1,\text{iterations});
\]
\[
equilibrium_m \% \text{displaying male equilibriums}
\]
\[
\% \text{plotting the female populations against time}
\]
\[
\text{plot}(x,S,'k^*',x,P,'r--',x,J_f,'g+',x,R,'b^');
\]
\[
\text{axis}([0 \text{ iterations } 0 N]);
\]
\[
\text{xlabel}('\text{Time... (Days)}');
\]
\[
\text{ylabel}('\text{Populations... across... time}');
\]
\[
\text{legend}('\text{Susceptible}', '\text{Prostitute}', 'Jailed', 'Rehabilitated');
\]
\[
\% \text{hold on;}
\]
\[
\text{figure};
\]
\[
\% \text{plotting the male populations against time}
\]
\[
\text{plot}(x,C_p,'k^*',x,G,'go',x,C_a,'r--',x,J_m,'b^');
\]
\[
\text{axis}([0 \text{ iterations } 0 M]);
\]
\[
\text{xlabel}('\text{Time... (Days)}');
\]
\[
\text{ylabel}('\text{Populations... across... time}');
\]
\[
\text{legend}('\text{Potential... Customer}', '\text{Abstainer}', 'Active... Customer', 'Jailed');
\]
\[
\text{test}(1,\text{iterations})
\]
\[
R_m = ((\eta_1(1,\text{iterations})*C_p(1,\text{iterations}))/((\omega(1,t-1) + \gamma_m(1,t-1) + \mu)) + \delta_m(((\gamma_m(1,t-1)) /((\omega(1,t-1) + \gamma_m(1,t-1) + \mu)) + \kappa_m)) /((\mu + \kappa_m))'),
\]
This appendix lists the MATLAB code used to simulate the modified model.

```
Listing C.1: Modified Model

clear all;
close all;

iterations = 20000;

%female model parameters
% beta  = 1/365;
    mu = 1/14600;
gamma_f = 0.0137/365;
delta_f = 0.5;
kappa_f = 0.15/365;
% rho = 3/365;

%male model parameters
lambda = 0.9;
% eta_1 = 30/365;
et_2  = 0.05/365; %was 0.05/365
gamma_m = 0.25/365;
%omega = 0.5/365000;
kappa_m = 15/365;
delta_m = 0.7;

%variable constants
beta_0 = .1/365;
k_m = 50/365;
rho_0 = 0.3/365;
et_4 = 0.4/365; %was 0.5/365
epsilon = 0.001;
```
C. Modified Model

\%sigma_m = 0.3/365;
\%sigma_f = 1.5/365;
omega_0 = 0.5/365; %was 0.5/365

%variable vectors
beta = zeros(1, iterations);
rho = zeros(1, iterations);
eta_1 = zeros(1, iterations);
%gamma_m = zeros(1, iterations);
%gamma_f = zeros(1, iterations);
omega = zeros(1, iterations);

%female model population vectors
S = zeros(1, iterations);
P = zeros(1, iterations);
J_f = zeros(1, iterations);
R = zeros(1, iterations);

%male model population vectors
C_p = zeros(1, iterations);
G = zeros(1, iterations);
C_a = zeros(1, iterations);
J_m = zeros(1, iterations);

%female model initial values
N = 1; %Total population
S(1,1) = 0.85;
P(1,1) = 0.1;
J_f(1,1) = 0.02;
R(1,1) = 0.03;

%male model initial values
M = 1; %Total population
C_p(1,1) = 0.55;
G(1,1) = 0.1;
C_a(1,1) = 0.349;
J_m(1,1) = 0.001;

%variable initial values
beta(1,1) = 1/365;
rho(1,1) = 3/365;
eta_1(1,1) = 0.5/365;
%gamma_m(1,1) = 0.25/365;
%gamma_f(1,1) = 1.37/365;
omega(1,1) = 0.5/365;
%female model individual components
birth_f = 1:iterations;
flow_SP = 1:iterations;
deadth_S = 1:iterations;
flow_PJ = 1:iterations;
deadth_P = 1:iterations;
flow_JP = 1:iterations;
flow_RP = 1:iterations;
deadth_J_f = 1:iterations;
flow_JR = 1:iterations;
deadth_R = 1:iterations;
kapJ_f = 1:iterations;

%male model individual components
birth_m = 1:iterations;
flow_Cp_Ca = 1:iterations;
flow_Ca_Cp = 1:iterations;
flow_J_Cp = 1:iterations;
deadth_Cp = 1:iterations;
flow_J_Ca = 1:iterations;
flow_Ca_J = 1:iterations;
deadth_Ca = 1:iterations;
kapJ_m = 1:iterations;
deadth_J_m = 1:iterations;
test = 1:iterations;
for t = 2:1:iterations;
%running the female model
S(1,t) = S(1,t-1) + mu*N - (beta(1,t-1))*(S(1,t-1)*P(1,t-1)/N) - mu*(S(1,t-1));
P(1,t) = P(1,t-1) + (beta(1,t-1))*(S(1,t-1)*P(1,t-1)/N) - (gamma_f + mu)*(P(1,t-1)) + (delta_f*kap_f)*((J_f(1,t-1))/(N));
J_f(1,t) = J_f(1,t-1) + (gamma_f)*P(1,t-1) - (kap_f + mu)*J_f(1,t-1);
R(1,t) = R(1,t-1) + (1-delta_f)*kap_f*(J_f(1,t-1)) - (rho*(1,t-1))*((R(1,t-1))*(P(1,t-1)))/(N) - mu*(R(1,t-1));
%running the male model
C. Modified Model

\begin{verbatim}
C_p(1, t) = C_p(1, t-1) + lambda*mu*M - ((eta_1(1, t-1))*C_p(1, t-1)) / M - ((eta_2*(C_p(1, t-1))*(G(1, t-1)))/M) + ((omega_1(1, t-1))*(C_a(1, t-1))) + ((1 - delta_m)*kappa_m*(J_m(1, t-1)) - (mu*(C_p(1, t-1)))
G(1, t) = G(1, t-1) + (1 - lambda)*mu*M + ((eta_2*(C_p(1, t-1)))
C_a(1, t) = C_a(1, t-1) + ((eta_1(1, t-1))*C_p(1, t-1)) / M + (delta_m*kappa_m*(J_m(1, t-1))) - (((gamma_m) + (omega(1, t-1)) + (mu))*(C_a(1, t-1)))
J_m(1, t) = J_m(1, t-1) + ((gamma_m)*(C_a(1, t-1))) - ((kappa_m + mu)*(J_m(1, t-1)))

%running the combined parameters
beta(1, t) = beta_0*k_m*((C_a(1, t-1))/P(1, t-1))
rho(1, t) = rho_0*k_m*((C_a(1, t-1))/P(1, t-1))
eta_1(1, t) = ((eta_m*(P(1, t-1))))/(P(1, t-1) + epsilon*N)

%gamma_m(1, t) = (sigma_m*(C_a(1, t-1)))/M;
%gamma_f(1, t) = (sigma_f*(P(1, t-1)))/N;
omega(1, t) = ((omega_0*(J_m(1, t-1)))/(C_a(1, t-1)) + (C_p(1, t-1)) + (J_m(1, t-1)))

%running the female indivudal components
birth_f(1, t) = mu*N;
flow_SP(1, t) = (beta(1, t-1))*(S(1, t-1)*P(1, t-1))/N;
death_S(1, t) = mu*(S(1, t-1))
flow_PJ(1, t) = (gamma_f)*P(1, t-1);
death_P(1, t) = (mu)*(P(1, t-1))
flow_JP(1, t) = (delta_f*kappa_f*(J_f(1, t-1)))
flow_RP(1, t) = (rho(1, t-1))*(R(1, t-1)*P(1, t-1))/N;
death_J_f(1, t) = mu*(J_f(1, t-1))
flow_JR(1, t) = (1 - delta_f)*kappa_f*(J_f(1, t-1))
death_R(1, t) = mu*(R(1, t-1))
kappa_J_f(1, t) = kappa_f*(J_f(1, t-1));

%running the female indivudal components
birth_m(1, t) = lambda*mu*M;
flow_Cp_Ca(1, t) = (((eta_1(1, t-1))*(C_p(1, t-1))*(C_a(1, t-1)))/M)
flow_Ca_Cp(1, t) = ((omega(1, t-1))*(C_a(1, t-1)))
flow_J_Cp(1, t) = ((1 - delta_m)*kappa_m*(J_m(1, t-1))
death_Cp(1, t) = (mu*(C_p(1, t-1)))
flow_J_Ca(1, t) = delta_m*kappa_m*(J_m(1, t-1))
flow_Ca_J(1, t) = (gamma_m)*(C_a(1, t-1));
\end{verbatim}
C. Modified Model

dead_Ca(1, t) = mu*(C_a(1, t-1));
kapJ_m(1, t) = kappa_m*(J_m(1, t-1));
dead_J_m(1, t) = mu*(J_m(1, t-1));
test(1, t) = ((C_p(1, t-1)) + C_a(1, t-1) + (J_m(1, t-1)) + G(1, t-1));
end
x = 1:iterations;
% storing female model equilibrium populations
equilibrium_f = 4:1;
equilibrium_f(1,1) = S(1, iterations);
equilibrium_f(2,1) = P(1, iterations);
equilibrium_f(3,1) = J_f(1, iterations);
equilibrium_f(4,1) = R(1, iterations);
equilibrium_f % displaying female equilibriums
% storing male model equilibrium populations
equilibrium_m = 4:1;
equilibrium_m(1,1) = C_p(1, iterations);
equilibrium_m(2,1) = G(1, iterations);
equilibrium_m(3,1) = C_a(1, iterations);
equilibrium_m(4,1) = J_m(1, iterations);
equilibrium_m % displaying male equilibriums
% plotting the female populations against time
plot(x, S, 'k--', x, P, '--r', x, J_f, 'g--', x, R, 'b--');
axis([0 iterations 0 N]);
xlabel('Time (Days)');
ylabel('Populations across time');
legend('Susceptible', 'Prostitute', 'Jailed', 'Rehabilitated');
% hold on;
figure;
% plotting the male populations against time
plot(x, C_p, 'k*', x, G, 'go', x, C_a, 'r--', x, J_m, 'b^');
axis([0 iterations 0 M]);
xlabel('Time (Days)');
ylabel('Populations across time');
legend('Potential Customer', 'Abstainer', 'Active Customer', 'Jailed');
test(1, iterations)
R_m = ((eta_1(1, iterations)*C_p(1, iterations)) / (omega(1, t-1) + gamma_m + mu)) + delta_m * ((gamma_m) / (omega(1, t-1) + gamma_m + mu)) * ((kappa_m) / (mu + kappa_m))'
APPENDIX D

TRAFFICKING MODEL

This appendix lists the MATLAB code used to simulate the trafficking model.

Listing D.1: Trafficking Model

```matlab
clear all;
close all;

iterations = 700000;

%female model parameters
% beta = 1/365;
    mu = 1/14600;
% gamma_f = 0.00137/365;
% delta_f = 0.5;
% kappa_f = 0.15/365;
% rho = 3/365;

%male model parameters
lambda = 0.9;
% eta_1 = 30/365;
    eta_2 = 0.05/365;
% gamma_m = 0.4/365;
% omega = 0.5/365000;
kappa_m = 15/365;
delta_m = 0.7;

%variable constants
beta_1 = 0.15/365;
beta_2 = 0.15/365;
k_m = 50/365;
rho_0 = 0.3/365;
etta_m = 0.4/365;
```
epsilon = 0.001;
%sigma_m = 0.3/365;
%sigma_f = 1.5/365;
omega_0 = 0.6/365;

%variable vectors
betaVP = zeros(1, iterations);
betaTP = zeros(1, iterations);
rho = zeros(1, iterations);
eta_1 = zeros(1, iterations);
%gamma_m = zeros(1, iterations);
%gamma_f = zeros(1, iterations);
omega = zeros(1, iterations);

%female model population vectors
S = zeros(1, iterations);
VP = zeros(1, iterations);
TP = zeros(1, iterations);
J_f = zeros(1, iterations);
R = zeros(1, iterations);

%male model population vectors
C_p = zeros(1, iterations);
G = zeros(1, iterations);
C_a = zeros(1, iterations);
J_m = zeros(1, iterations);

%female model initial values
N = 1; %Total population
S(1,1) = 0.80;
VP(1,1) = 0.05;
TP(1,1) = 0.1;
J_f(1,1) = 0.02;
R(1,1) = 0.03;

%male model initial values
M = 1; %Total population
C_p(1,1) = 0.55;
G(1,1) = 0.1;
C_a(1,1) = 0.349;
J_m(1,1) = 0.001;

%variable initial values
betaVP(1,1) = 1/365;
betaTP(1,1) = 2/365;
%running the combined parametres

\[ \begin{align*}
\text{betaTP}(1, t) &= \beta_1 k_m ((C_a(1, t - 1))/((C_a(1, t - 1)) + (C_p(1, t - 1))))) \\
\text{betaVP}(1, t) &= \beta_2 k_m ((C_a(1, t - 1))/((C_a(1, t - 1)) + (C_p(1, t - 1)))))
\end{align*} \]

\[
\text{J}_m(1, t) = J_m(1, t - 1) + \left( (\eta_1 - \eta_2) + \left( \sum_{\text{iterations}} \right) \right)
\]

\[
\text{J}_f(1, t) = J_f(1, t - 1) + \left( \gamma_m - \gamma_f + \left( \sum_{\text{iterations}} \right) \right)
\]

\[
\text{C}_p(1, t) = C_p(1, t - 1) + \lambda \mu M - \left( (\eta_1 - \eta_2) + \left( \sum_{\text{iterations}} \right) \right)
\]

\[
\text{G}(1, t) = G(1, t - 1) + \left( \mu - \left( \sum_{\text{iterations}} \right) \right)
\]

\[
\text{C}_a(1, t) = C_a(1, t - 1) + \left( (\eta_1 - \eta_2) + \left( \sum_{\text{iterations}} \right) \right)
\]

\[
\text{J}_m(1, t) = J_m(1, t - 1) + \left( \gamma_m - \gamma_f + \left( \sum_{\text{iterations}} \right) \right)
\]

\[
\text{J}_f(1, t) = J_f(1, t - 1) + \left( \gamma_m - \gamma_f + \left( \sum_{\text{iterations}} \right) \right)
\]

\[
\text{test}_m = 1: \text{iterations} \\
\text{test}_f = 1: \text{iterations}
\]

\[
\text{for} \ t = 2: \text{iterations} \\
\]

%running the female model

\[
\text{S}(1, t) = S(1, t - 1) + \mu N - (\beta_{VP}(1, t - 1) \times (S(1, t - 1) \times VP(1, t - 1)) \div N) - \left( (\beta_{TP}(1, t - 1) \times (S(1, t - 1) \times TP(1, t - 1)) \div N \right) - \mu \times (S(1, t - 1))
\]

\[
\text{TP}(1, t) = TP(1, t - 1) + ((\beta_{TP}(1, t - 1) \times (S(1, t - 1) \times TP(1, t - 1)) \div N) - (\gamma_m + \mu) \times (TP(1, t - 1))
\]

\[
\text{J}_f(1, t) = J_f(1, t - 1) + ((\gamma_f - \gamma_m) + \left( \sum_{\text{iterations}} \right)) \times (J_f(1, t - 1))
\]

\[
\text{R}(1, t) = R(1, t - 1) + (1 - \delta_f) \times \kappa_f \times (J_f(1, t - 1)) - ((\gamma_m + \mu) \times (R(1, t - 1))
\]

%running the male model

\[
\text{C}_p(1, t) = C_p(1, t - 1) + \lambda \mu M - ((\eta_1 - \eta_2) \times (C_p(1, t - 1)) \times (C_a(1, t - 1)) \div M) - ((\eta_2 - (C_p(1, t - 1)) \times (G(1, t - 1)) \div M) + ((\gamma_m - \gamma_f) \times (C_a(1, t - 1)) \div M) - \left( \sum_{\text{iterations}} \right)
\]

\[
\text{G}(1, t) = G(1, t - 1) + (1 - \lambda) \times \mu M + ((\eta_2 - (C_p(1, t - 1)) \times (G(1, t - 1)) \div M)
\]

\[
\text{C}_a(1, t) = C_a(1, t - 1) + ((\eta_1 - \eta_2) \times (C_p(1, t - 1)) \times (C_a(1, t - 1)) \div M) + ((\gamma_m - \gamma_f) \times (C_a(1, t - 1)) \div M) - (\left( \sum_{\text{iterations}} \right)
\]

\[
\text{J}_m(1, t) = J_m(1, t - 1) + ((\gamma_m - \gamma_f) \times (C_a(1, t - 1)) - (\kappa_m + \mu) \times (J_m(1, t - 1))
\]
\[ \rho(1, t) = \rho_0 \ast k_m \ast \frac{(C_a(1, t-1))}{(VP(1, t-1) + TP(1, t-1))}; \]
\[ \eta_1(1, t) = \frac{(\eta_m \ast (VP(1, t-1) + TP(1, t-1)))}{(VP(1, t-1) + TP(1, t-1)) + \epsilon \ast N)}; \]
\[ \gamma_m(1, t) = \frac{(\sigma_m \ast (C_a(1, t-1)))}{M}; \]
\[ \gamma_f(1, t) = \frac{(\sigma_f \ast (P(1, t-1)))}{N}; \]
\[ \omega(1, t) = \frac{(\omega_0 \ast (J_m(1, t-1)))}{(C_a(1, t-1) + C_p(1, t-1) + J_m(1, t-1))}; \]
\[ \text{test}_m(1, t) = (C_p(1, t-1) + C_a(1, t-1) + J_m(1, t-1) + G(1, t-1)); \]
\[ \text{test}_f(1, t) = (S(1, t-1) + VP(1, t-1) + TP(1, t-1) + J_f(1, t-1) + R(1, t-1)); \]
end

\[ x = 1:\text{iterations}; \]

% storing female model equilibrium populations
equilibrium_f = 5:1;
equilibrium_f(1, 1) = S(1, iterations);
equilibrium_f(2, 1) = VP(1, iterations);
equilibrium_f(3, 1) = TP(1, iterations);
equilibrium_f(4, 1) = J_f(1, iterations);
equilibrium_f(5, 1) = R(1, iterations);
equilibrium_f % displaying female equilibriums

% storing male model equilibrium populations
equilibrium_m = 4:1;
equilibrium_m(1, 1) = C_p(1, iterations);
equilibrium_m(2, 1) = G(1, iterations);
equilibrium_m(3, 1) = C_a(1, iterations);
equilibrium_m(4, 1) = J_m(1, iterations);
equilibrium_m % displaying male equilibriums

% plotting the female populations against time
plot(x, S, 'm', x, VP, 'r', x, TP, 'k', x, J_f, 'g', x, R, 'b');
axis([0 iterations 0 1]);
xlabel('Time (Days)');
ylabel('Populations across time');
legend('Susceptible', 'Voluntary', 'Trafficked', 'Jailed', 'Rehabilitated');
% hold on;
figure;
D. Trafficking Model

```matlab
% plotting the male populations against time
plot(x,C_p,'k-',x,G,'g-',x,C_a,'r-',x,J_m,'b-');
axis([0 iterations 0 1]);
xlabel('Time (Days)');
ylabel('Populations across time');
legend('Potential Customer','Abstainer','Active Customer','Jailed');
test_m(1,iterations)
test_f(1,iterations)

R_m = ((eta_1(1,iterations)*C_p(1,iterations))/(omega(1, iterations) + gamma_m + mu)) + delta_m*((gamma_m)/(omega (1,iterations) + gamma_m + mu)) *((kappa_m)/(mu + kappa_m))'
```
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